

OVERTONE AND MULTI-MODE RR LYRAE STARS IN THE GLOBULAR CLUSTER M3

J. JURCSIK¹, P. SMITOLA¹, G. HAJDU^{2,3}, Á. SÓDOR¹, J. NUSPL¹, K. KOLENBERG^{4,5}, G. FÜRÉSZ⁶, A. MOÓR¹, E. KUN^{7,8}, A. PÁL¹, J. BAKOS¹, J. KELEMEN¹, T. KOVÁCS¹, L. KRISKOVICS¹, K. SÁRNECZKY¹, T. SZALAI⁹, A. SZING¹⁰, K. VIDA¹¹Konkoly Observatory of the Hungarian Academy of Sciences, H-1525 Budapest PO Box 67, Hungary²Instituto de Astrofísica, Pontificia Universidad Católica de Chile, Av. Vicuña Mackenna 4860, 782-0436 Macul, Santiago, Chile³Millennium Institute of Astrophysics, Av. Vicuña Mackenna 4860, 782-0436 Macul, Santiago, Chile⁴Institute of Astronomy, KU Leuven, Celestijnenlaan 200D, 3001 Heverlee, Belgium⁵Harvard-Smithsonian Center for Astrophysics, 60 Garden Street, Cambridge MA 02138, USA⁶MIT Kavli Institute for Astrophysics and Space Research 77 Mass Ave 37-515, Cambridge, MA, 02139⁷Department of Theoretical Physics, University of Szeged, H-6720 Szeged, Tisza Lajos krt 84-86, Hungary⁸Department of Experimental Physics and Astronomical Observatory, University of Szeged, H-6720 Szeged, Dóm tér 9, Hungary⁹Department of Optics and Quantum Electronics, University of Szeged, H-6720 Szeged, Dóm tér 9, Hungary and¹⁰Baja Observatory, University of Szeged, 6500 Baja, KT: 766, Hungary*Draft version April 24, 2015*

ABSTRACT

The overtone and multi-mode RR Lyrae stars in the globular cluster M3 are studied using a 200-d long, B , V and I_C time-series photometry obtained in 2012. 70% of the 52 overtone variables observed show some kind of multi-periodicity (additional frequency at $f_{0.61} = f_{10}/0.61$ frequency ratio, Blazhko effect, double/multi-mode pulsation, period doubling). A signal at 0.587 frequency ratio to the fundamental-mode frequency is detected in the double-mode star, V13, which may be identified as the second radial overtone mode. If this mode-identification is correct, than V13 is the first RR Lyrae star showing triple-mode pulsation of the first three radial modes. Either the Blazhko effect or the $f_{0.61}$ frequency (or both of these phenomena) appear in 7 double-mode stars. The P_{10}/P_F period ratio of RRd stars showing the Blazhko effect are anomalous. A displacement of the main frequency component at the fundamental-mode with the value of modulation frequency (or its half) is detected in three Blazhko RRd stars parallel with the appearance of the overtone-mode pulsation. The $f_{0.61}$ frequency appears in RRc stars that lie at the blue side of the double-mode region and in RRd stars, raising the suspicion that its occurrence may be connected to double-mode pulsation. The changes of the Blazhko and double-mode properties of the stars are also reviewed using the recent and archive photometric data.

Subject headings: Galaxy: globular clusters: individual: M3 – stars: horizontal-branch – stars: oscillations – stars: variables: RR Lyr

1. INTRODUCTION

Studies of cluster variables (RR Lyrae stars) date back to the end of the 19th century. The large number of the variables discovered in galactic and extragalactic globular clusters (GCs) provided a decisive support to specify the main physical properties and the evolutionary status of RR Lyrae stars. Because of the large distance ($d > 6$ kpc) and the relatively small absolute dimension (total radius, $R_t < 50$ pc) of GCs, the apparent magnitudes of the cluster members reflect the distribution of their absolute magnitudes with 1 – 2% accuracy (supposing that the foreground/intra-cluster reddening is homogeneous). The advantage that the relative values of the physical properties of GC stars can be determined more accurately than for any other stellar population makes GC studies very prosperous.

While numerous studies of fundamental-mode RR Lyrae stars (RRab) have appeared year by year in the literature, this is not the case for the overtone (RRc) variables. As RRc stars are relatively rare objects and the similarity of their light-curves to W UMa binaries makes even the target selection dubious, hence only a few field RRc stars have been the subject of detailed photometric and/or spectroscopic studies. The photometric data of

GCs showed that about 30% of their RR Lyrae populations are overtone and double-mode (RRd) pulsators (Clement et al. 2001). RRd stars have always been in the focus of the interest because of their astrophysical importance, and the discovery of peculiar features in the pulsations of RRc stars (Moskalik 2014; Moskalik et al. 2015; Netzel et al. 2015; Smolec 2015a,b, and references therein) has attracted a special attention to overtone RR Lyrae stars recently. Higher order or multiple pulsations together with modulation or period doubling make these stars to unique targets of theoretical investigations. The complex and interrelated dynamical features can give strong constraints on non-linear physics governing their variability and pose a new challenge to further theoretical work, providing a test bed for improved models.

Thanks to the new data secured by space missions (CoRoT, Kepler: Szabó et al. 2014; Szabó 2014) and ground-based projects (Jurcsik 2009; Le Borgne et al. 2012), a lot of new results were published in the recent years concerning the Blazhko effect; the amplitude and/or phase modulation of the pulsation of RR Lyrae stars. Despite these observational results and the accompanying theoretical efforts revealing the genuine origin of the phenomenon has eluded explanation yet. The modulation was detected in about 50% of the RRab stars (Jurcsik et al. 2009; Benkő et al. 2014), but no reliable

TABLE 1

B , V , I_C CCD OBSERVATIONS OF M3 VARIABLES PUBLISHED IN THIS PAPER. THE COMPLETE TABLE IS GIVEN IN THE ELECTRONIC EDITION.

Star ^a	HJD–2400000 ^b	Mag	Band
V012	55935.64720	16.088	B
V012	55937.69026	15.583	B
V012	55937.70074	15.636	B
...

^aIdentification according to the catalogue of Samus et al. (2009)

^bThe HJDs represent the times of mid-exposure.

TABLE 2

FLUX DATA OF M3 VARIABLES WITH UNCERTAIN MAGNITUDES IN THE REFERENCE FRAME IN B BAND. THE COMPLETE TABLE IS GIVEN IN THE ELECTRONIC EDITION.

Star ^a	HJD–2400000 ^b	Flux ^c
V129	55935.64720	15583.9
V129	55937.69026	16631.5
V129	55937.70074	15274.6
...

^aIdentification according to the catalogue of Samus et al. (2009)

^bThe HJDs represent the times of mid-exposure.

^c B band flux differences of ISIS photometry (Alard 2000).

Blazhko statistics exists for overtone variables.

One of the most intriguing new question of overtone RR Lyrae and Cepheid stars is the recent discovery of a peculiar frequency at 0.61 frequency ratio in many of these variables (Soszyński et al. 2010; Dziembowski 2012; Moskalik et al. 2015, and references therein). The interpretation of this strange signal is hard in the recent theoretical framework because of its unknown excitation mechanism.

M3 is one of GCs with a large RR Lyrae population that was investigated in many photometric studies previously. However, even the most extended CCD observations [e.g. Kaluzny et al. (1998, K98), Corwin & Carney (2001, C01), Hartman et al. (2005, H05), Benkő et al. (2006, B06), and Jurcsik et al. (2012a, J12)] were not long and dense enough and/or used only one filter, therefore our knowledge on the multi-mode and Blazhko properties of M3 variables is still incomplete. The present article gives an in-depth study of the overtone and double-mode RR Lyrae stars of M3, with full details on their modulations and multi-mode properties, as well as on the long-term changes of the light-curve variations based on an extended new multi-color time-series photometry of the cluster and on archive data. Some interesting features of the Blazhko and double-mode properties of overtone variables using the new data set published here were already published in Jurcsik et al. (2014).

2. THE PHOTOMETRIC DATA: OBSERVATIONS, DATA PROCESSING AND ANALYSIS

An extended photometric campaign was organized to observe M3 with the Konkoly Observatory 90/60 cm Schmidt telescope (Piszkéstető) equipped with a 4096x4096 Apogee CCD camera and Bessel BVI_C filters. About 1000 measurements were obtained in each band between January and July in 2012. The standard calibration processes of the CCD frames were performed using

IRAF/CCDRED¹ tasks. The data were not corrected for the nonlinearity (less than 2%) of the CCD system. As a consequence of the nonlinearity, linear combinations of independent frequencies may appear in the data as artifacts with mmag amplitude, and the measured amplitudes may be uncertain up to 1 – 2%. Both aperture (IRAF/DAOPHOT/PHOT) and Image Subtraction Method (ISIS, Alard 2000) photometry of the variables were performed. The light curves were transformed to the standard BVI_C system using 23 bright, photometrically stable standard stars, covering a wide range in color, from the list of Stetson (2000).

Time-series photometry of 52 overtone and double-mode RR Lyrae was obtained. The flux-magnitude transformation of the results of the ISIS photometry needs accurate magnitudes on the reference frames. The PSF photometry (using IRAF/DAOPHOT/ALLSTAR) on the reference frames was performed very circumspectly; the neighboring stars of each variable were checked on SDSS images². The BVI_C calibration of the light curves were accurate for 28 stars (Fig. 1), but it remained uncertain because of crowding for 24 variables. Besides of the pulsation period, additional properties of the light curves were detected in some of these non-calibrated flux time series. Therefore, the magnitude transformation of the B -filter flux curves of these 24 stars was also performed (Fig. 2) with reference magnitude values that yielded mean magnitudes matching the horizontal-branch magnitude of the cluster, just for the visualization of the light curves. The time-series photometry obtained using the image subtraction method is sensitive to the contamination of only nearby variables. The frequencies of close variables in the neighborhood appeared with low amplitude in the spectrum of some of the variables with non-calibrated light curves, but these signals were removed from the data. No flux curve could be derived for five overtone variables (V152, V155, V178, V266, and V269) because their ISIS photometry gave unreliable results due to blending with another variable. The CCD time series of the variables are available from the electronic edition of this article; Table 1 and Table 2 give samples of the magnitude calibrated B , V , I_C , and the B -filter flux data regarding their form and content. These data are referred as D12 in the paper.

The light curves were analyzed using the program packages MUFRAN (Kolláth 1990) and LCfit (Sódor et al. 2012). Independent frequencies were selected in successive prewhitening steps if their amplitudes were larger than 4σ at least in 2 bands. For linear-combination terms a less strict criterion was used.

The long-term behavior and the changes of some special properties of the stars were checked using the CCD data published in K98, C01, H05, B06 and J12 and in the photographic data collected in J12. All the previous photometric data are magnitude homogenized and merged.

3. THE LIGHT CURVES, VARIABLE TYPES

¹ IRAF is distributed by the National Optical Astronomy Observatories, which are operated by the Association of Universities for Research in Astronomy, Inc., under cooperative agreement with the National Science Foundation.

² <http://www.sdss.org/>.

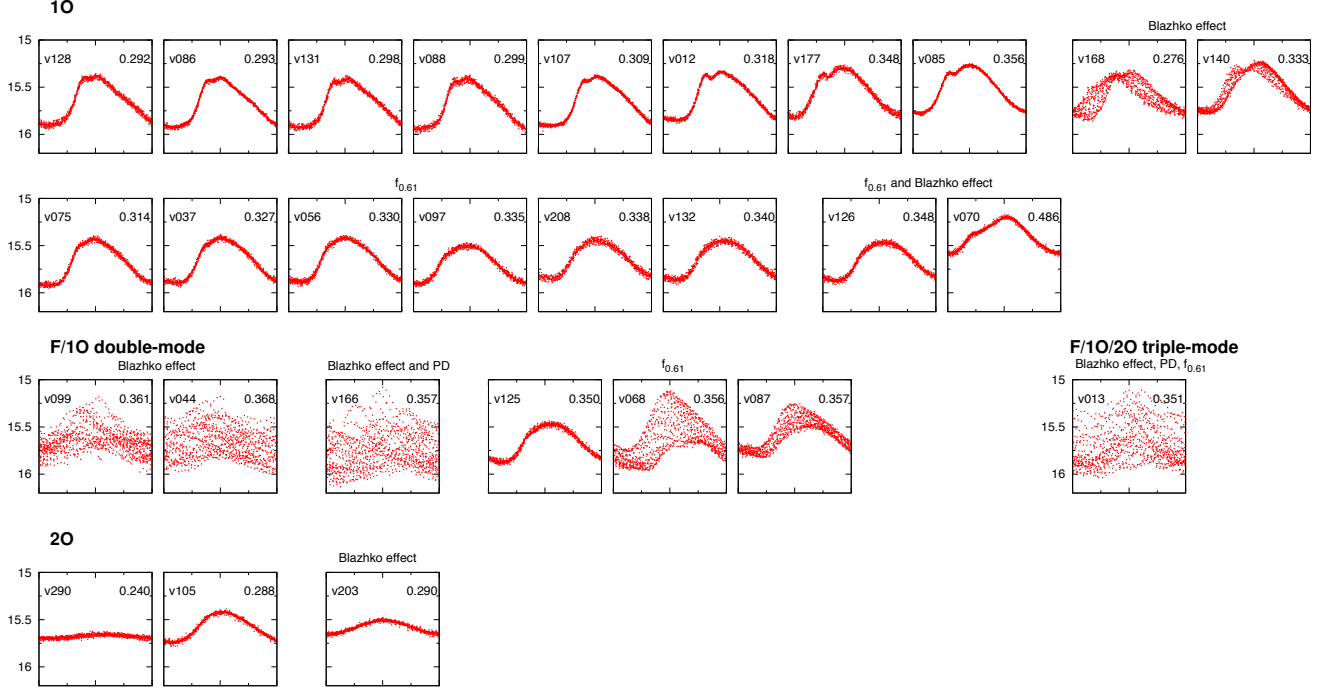


FIG. 1.— V light curves of overtone and double-mode variables in M3 phased with the overtone-mode period. The names and periods (days) of the variables are given in the top-left- and top-right-side of each panel.

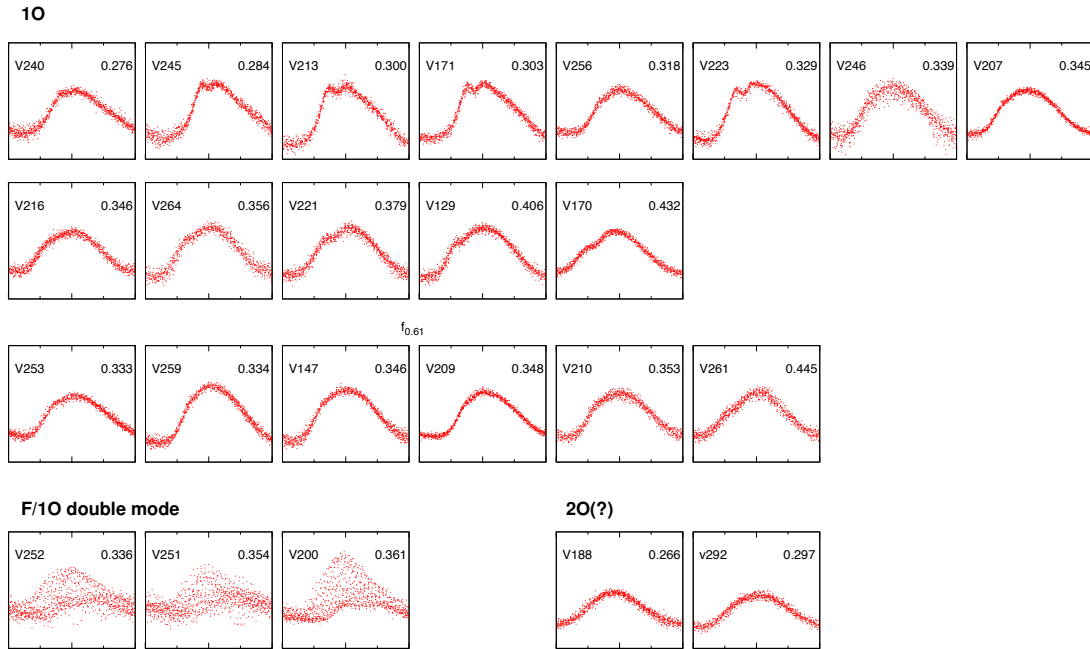


FIG. 2.— Non-calibrated B -filter light curves of overtone and double-mode variables in M3 phased with the overtone-mode period. The ISIS fluxes of the variables with uncertain magnitudes on the reference frame have been transformed to magnitudes matching the mean brightness of the horizontal branch, however, the amplitudes of these light curves are uncertain.

TABLE 3
SUMMARY OF THE PHOTOMETRIC PROPERTIES OF THE OVERTONE-MODE VARIABLES IN M3.

Star ^a	mode ^b	period d	$\langle B \rangle^c$ mag	$\langle B \rangle^c$ mag	$\langle V \rangle$ mag	$\langle V \rangle$ mag	$\langle I_C \rangle$ mag	$\langle I_C \rangle$ mag	A_B^d mag	A_V^d mag	$A_{I_C}^d$ mag	additional feature ^e
V12	1O	0.31786	15.829	15.802	15.600	15.584	15.264	15.258	0.65	0.51	0.32	
V37	1O	0.32663	15.909	15.885	15.664	15.650	15.311	15.306	0.64	0.48	0.30	$f_{0.61}$
V56	1O	0.32960	15.908	15.884	15.657	15.643	15.278	15.272	0.62	0.47	0.29	$f_{0.61}$
V70	1O	0.48607	15.691	15.677	15.398	15.391	14.963	14.960	0.53-0.49	0.40-0.38	0.25-0.24	$f_{0.61}$ Bl
V75	1O	0.31408	15.924	15.899	15.686	15.672	15.306	15.300	0.63	0.49	0.30	$f_{0.61}$
V85	1O	0.35581	15.746	15.720	15.521	15.505	15.190	15.184	0.65	0.51	0.32	
V86	1O	0.29266	15.914	15.886	15.679	15.663	15.367	15.361	0.69	0.54	0.33	
V88	1O	0.29875	15.923	15.893	15.693	15.675	15.369	15.362	0.70	0.54	0.32	
V97	1O	0.33493	15.973	15.955	15.699	15.689	15.329	15.325	0.53	0.40	0.26	$f_{0.61}$
V105	2O	0.28775	15.787	15.776	15.581	15.576	15.299	15.297	0.42	0.32	0.20	
V107	1O	0.30903	15.896	15.868	15.663	15.646	15.327	15.320	0.68	0.52	0.33	
V126	1O	0.34841	15.935	15.917	15.665	15.655	15.284	15.280	0.60-0.53	0.44-0.40	0.30-0.26	$f_{0.61}$ Bl
V128	1O	0.29204	15.894	15.866	15.657	15.641	—	—	0.69	0.52	—	
V129	1O	0.40604										
V131	1O	0.29769	15.909	15.882	15.684	15.668	15.399	15.392	0.67	0.52	0.34	
V132	1O	0.33986	15.909	15.892	15.653	15.643	15.259	15.247	0.53	0.41	0.25	$f_{0.61}$
V140	1O	0.33316	15.743	15.722	15.515	15.502	15.155	15.150	0.69-0.60	0.53-0.45	0.32-0.27	Bl
V147*	1O	0.34647										$f_{0.61}$
V168	1O	0.27594	15.803	15.788	15.588	15.580	15.345	15.342	0.65-0.46	0.48-0.35	0.32-0.23	Bl
V170*	1O	0.43238										
V171*	1O	0.30328										
V177	1O	0.34836	15.763	15.735	15.552	15.536	15.263	15.257	0.67	0.52	0.33	
V188*	2O:	0.26651										
V203	2O	0.28979	15.766	15.764	15.581	15.580	15.296	15.296	0.21-0.17	0.17-0.12	0.09-0.08	Bl
V207	1O	0.34530										
V208	1O	0.33838	15.923	15.905	15.647	15.638	15.266	15.262	0.55	0.40	0.26	$f_{0.61}$
V209	1O	0.34829										$f_{0.61}$
V210	1O	0.35295										$f_{0.61}$
V213*	1O	0.30001										
V216	1O	0.34648										
V221	1O	0.37878										
V223	1O	0.32921										
V240	1O	0.27601										
V245*	1O	0.28403										
V246*	1O	0.33916										
V253*	1O	0.33260										$f_{0.61}$
V256	1O	0.31806										
V259	1O	0.33351										$f_{0.61}$
V261*	1O	0.44473										$f_{0.61}$
V264*	1O	0.35646										
V290	2O	0.24041	15.861	15.861	15.679	15.679	15.415	15.415	0.05	0.04	0.03	
V292	2O:	0.29654										

^a Asterisk denotes stars with photometry contaminated by signals of nearby variables.

^b 1O: 1st overtone; 2O: 2nd overtone.

^c $\langle \rangle$ and $()$ denote magnitude and intensity averaged mean magnitudes, respectively.

^d Peak to peak amplitudes. Maximum and minimum amplitudes stars are given for Blazhko.

^e $f_{0.61}$: frequency at $f_{1O}/f_{0.61} \approx 0.61 - 0.62$; Bl: Blazhko effect.

TABLE 4
SUMMARY OF THE PHOTOMETRIC PROPERTIES OF DOUBLE/TRIPLE-MODE VARIABLES IN M3.

Star ^a	mode ^b	period d	$\langle B \rangle^c$ mag	$\langle B \rangle^c$ mag	$\langle V \rangle^c$ mag	$\langle V \rangle^c$ mag	$\langle I_C \rangle^c$ mag	$\langle I_C \rangle^c$ mag	A_B^d mag	A_V^d mag	$A_{I_C}^d$ mag	R^e	additional feature ^f	
V13	1O	0.35072	15.986	15.953	15.690	15.673	15.279	15.271	0.38-0.11	0.29-0.09	0.19-0.05		$f_{0.61}$	Bl
	F	0.47950							0.97-0.40	0.74-0.30	0.50-0.20			Bl
	2O	0.28160							0.020	0.019	0.010			PD
V44	1O	0.36812	15.951	15.928	15.659	15.646	15.220	15.215	0.25-0.14	0.19-0.11	0.12-0.07			Bl
	F	0.50377							0.87-0.34	0.62-0.24	0.40-0.19			Bl
V68	1O	0.35597	15.942	15.909	15.653	15.635	15.237	15.230	0.55	0.40	0.25		$f_{0.61}^g$	
	F	0.47850							0.48	0.35	0.22	0.87		
V87	1O	0.35749	15.871	15.853	15.583	15.573	15.168	15.164	0.49	0.37	0.23		$f_{0.61}$	
	F	0.48017							0.21	0.16	0.11	0.4		
V99	1O	0.3611	15.940	15.924	15.653	15.644	15.223	15.220	0.52-0.13	0.39-0.10	0.26-0.06			Bl
	F	0.4821							0.44-0.20	0.38-0.15	0.24-0.10			Bl
V125	1O	0.34982	15.939	15.922	15.665	15.656	15.262	15.258	0.54	0.42	0.26	0.02	$f_{0.61}$	
	F	0.4709							0.012	0.012	0.005			
V166	1O	0.35672	16.034	16.007	15.753	15.736	15.333	15.326	0.29-0.17	0.23-0.14	0.14-0.08			Bl
	F	0.48504							0.99-0.36	0.74-0.28	0.47-0.18			Bl PD
V200	1O	0.36102												
	F	0.48530										1.1		
V251*	1O	0.3542												
	F	0.4762										0.9		
V252*	1O	0.3361												
	F	0.4530										0.95		

^a Asterisk denotes stars with photometry contaminated by signals of nearby variables.

^b F: fundamental mode; 1O:1st overtone; 2O:2nd overtone.

^c $\langle \rangle$ and $()$ denote magnitude and intensity averaged mean magnitudes, respectively.

^d Peak to peak amplitudes of data prewhitened for the other mode and the linear-combination terms. Maximum and minimum amplitudes are given for Blazhko stars.

^e Amplitude ratio relative to the amplitude of the 1O mode in B band. Italics are for the non-calibrated data, transformed to magnitude with an uncertain zero point.

^f $f_{0.61}$: frequency at $f_{1O}/f_{0.61} \approx 0.61 - 0.62$; Bl: Blazhko effect; PD:period doubling.

^g the $f_{0.61}$ frequency is detected only in the V data.

Table 3 and Table 4 summarize the main characteristics of the light curves of the overtone and the double- (multi-)mode variables, respectively, based on the D12 data. Pulsation mode, period, magnitude-, and intensity-averaged B , V , I_C brightnesses, amplitudes, and remarks on any additional property are listed in the columns. The amplitude-ratio of the radial modes in B band is also given for non-Blazhko RRd stars in Table 4. The stars for which the photometry is contaminated by signals from nearby (within 6'') variable stars are denoted by asterisk in Tables 3 and 4.

The panels in Fig. 3 show the location of the stars on the V -period, $A(B)$ -period, $V-(B-V)$ and $(V-I)-(B-V)$ planes. The double-mode (red triangle and diamond), first overtone (blue circle) and second-overtone (black square) stars are located in separated groups with some overlapping in each plot. The extra properties of the stars, the Blazhko effect and the $f_{0.61}$ additional frequency are also indicated in the figure.

The Fourier amplitudes, amplitude ratios and phase differences of the V light curves are shown in Fig. 4. The same symbols are used for distinguishing stars with different properties as in Fig. 3. The location of different-type variables in Figs. 3 and 4 are discussed in the next corresponding sections.

3.1. Multi-mode pulsations, additional frequencies

The light curves of more than half of the studied stars are not stable; their Fourier spectra show extra frequency components. For example, an additional fre-

quency component appears in 18 stars at the frequency ratio $f_{1O}/f_{0.61} \approx 0.61 - 0.62$ (RR_{0.61} stars). Period doubling (PD) – manifested as signal at half-integer frequency ratio to the original one – is detected in two cases (V13 and V166). When the additional frequency appears close to the overtone frequency (and its harmonics), the Blazhko effect (Bl) is supposed to be observed, even if only one close frequency component is detected (V126, V166, and V203). Though, alternatively, the additional signal may correspond to a non-radial mode in these latter cases.

Ten double-mode variables are in the sample, one of them is, in fact, a triple-mode star. Besides the fundamental (F) and the first-overtone (1O) modes, the second-overtone mode (2O) [the identification is based on the $P/P_F = 0.587$ period ratio] also appears in the spectrum of V13. If the 2O identification of the third mode is correct, V13 is the first RR Lyrae star showing pulsation with the first three radial modes simultaneously.

The occurrence of the different types of phenomena are frequently interlaced. The most complex variation is observed in the triple-mode star, V13, which shows Blazhko effect of the F and 1O radial modes, period doubling of the 2O mode and the $f_{0.61}$ component as well. About half of the RRd stars show the Blazhko effect and their P_{1O}/P_F period ratios are anomalous.

In contrast with the fast and irregular period changes often detected in RRc stars, the period changes of overtone variables in M3 are marginal or modest. Irregular,

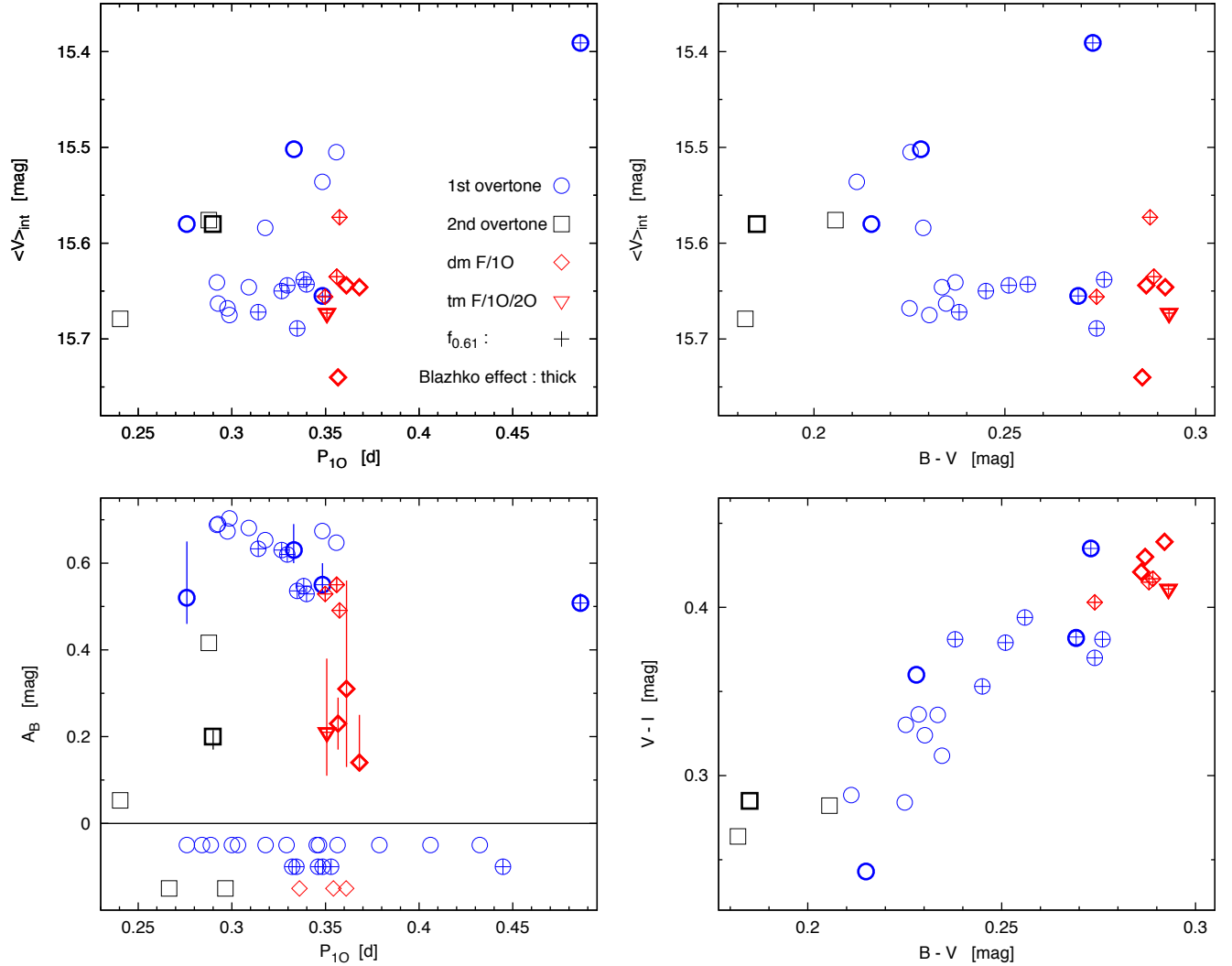


FIG. 3.— Intensity-averaged mean V magnitudes and peak-to-peak B amplitudes vs period of overtone variables are plotted on the left-side panels. 1O, 2O variables, F/1O stars and the F/1O/2O triple-mode variable, V13, are denoted by different symbols. The appearance of the Blazhko effect and the additional frequency component at $f_{10}/f_{0.61} \approx 0.61 - 0.62$ ratio is also indicated. The period distribution of stars with uncertain flux-magnitude transformation is shown in the bottom part of the $A(B) - P_{10}$ panel. For Blazhko stars the full range of the amplitude variation is indicated by a vertical line. The right-hand panels show intensity-averaged mean V magnitude (top) and $V - I$ color (bottom) vs $B - V$.

rapid period changes were detected only of two RRC stars (V12 and V70) and of some RRd stars in the period-change study of the RR Lyrae stars of M3 (J12). Therefore, the possibility that the detected additional features were the artifacts of period changes in a 200-d long time-series photometry can be excluded in most of the cases. The possible role of the period changes of V70 is discussed in Section 6.1, and the period changes of double-mode stars are summarized in Section 7.

3.2. Evolved and second-overtone variables

The overtone variables (V70, V170, and V261) with the longest periods are close to the short-end of the period range of the fundamental-mode variables ($P = 0.40 - 0.45$ d) and have light curves showing anomalous shape. Their rising branch is longer than of typical

ones and the bump appears at the middle of it, while that generally precedes maximum light in normal RRC stars slightly. Therefore, the RRC status of these highly evolved variables might be questionable. However, two of these stars (V70 and V261) show a secondary periodicity with 0.61 period ratio. The fact, that this type of multi-periodicity appears in many other RRC stars in M3, and exclusively in first-overtone variables according to other studies (see details in Section 4) supports the RRC status of these stars.

The identification of three 2O variables (V105, V203, and V290) is based on their short period, small-amplitude, smooth light curves and extreme color indices. Though its period and amplitude may indicate 2O status of V168 ($P = 0.276$ d) too, as this star shows

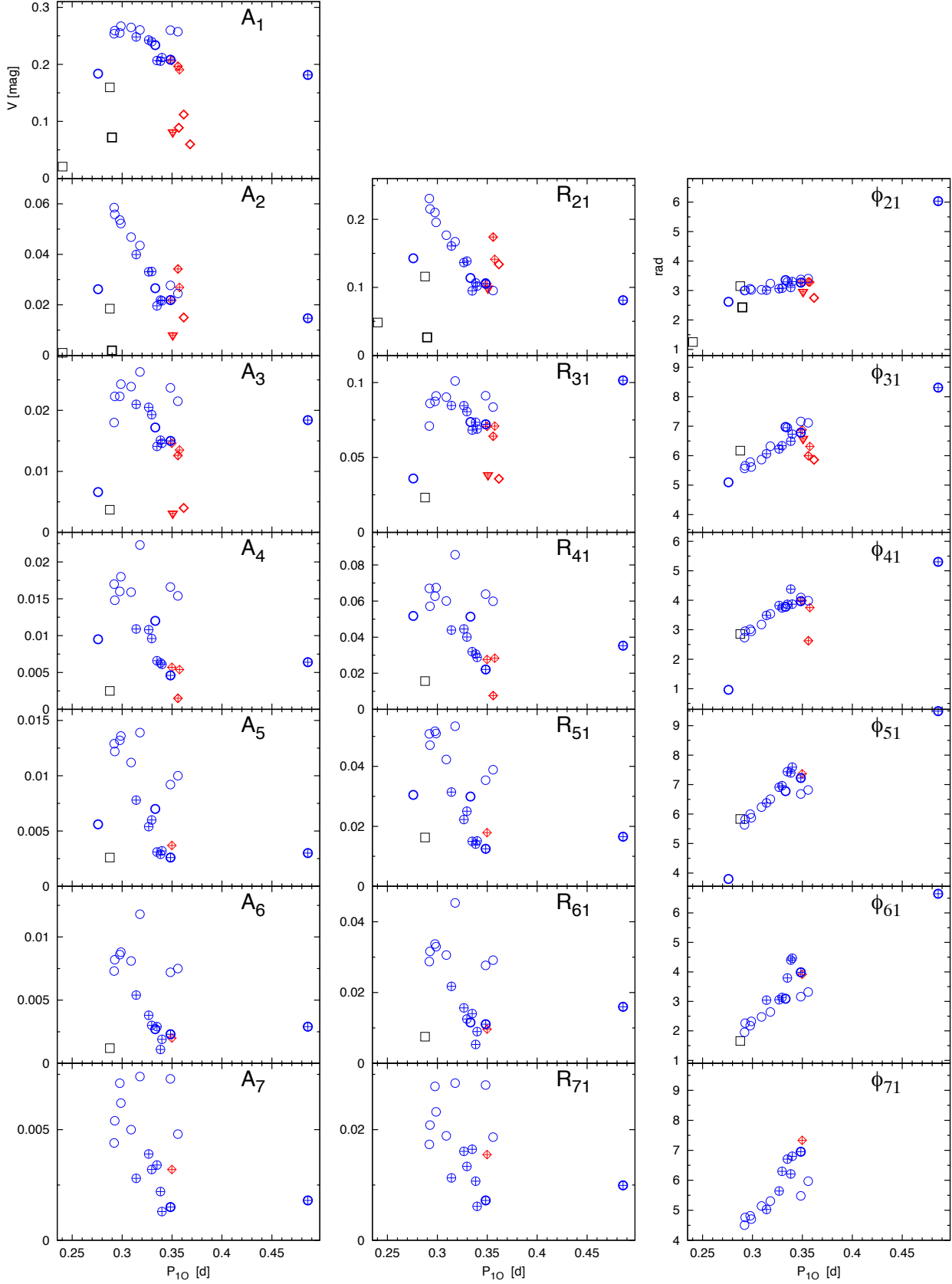


FIG. 4.— Fourier amplitudes (A_k), amplitude ratios (R_{k1}) and phase differences (ϕ_{k1}) of the V light curves of overtone variables and the overtone mode of double-mode stars are shown. The symbols are the same as in Fig. 3. Low-amplitude variables (second overtone, double-mode stars) do not appear in each plot, because they have detectable amplitudes only in the lowest harmonic orders.

TABLE 5
PARAMETERS OF THE $f_{0.61}$ FREQUENCY COMPONENTS.

Star	f_{1O} d^{-1}	$f_{0.61}$ d^{-1}	$f_{1O}/f_{0.61}$	$A_{0.61}^a$ mag	$A_{0.61}/A_{1O}^b$	Data ^c
V13	2.8513	4.6457	0.6137	0.006	0.075	D12
V37	3.0615	4.977	0.6151	0.006	0.025	D12
		4.965	0.6166	0.004	0.018	D12
	3.0615	4.9660	0.6165	0.007	0.028	H05+B06
V56	3.0340	4.944	0.6137	0.009	0.036	D12
		4.911	0.6137	0.005	0.019	D12
	3.0341	4.9475	0.6133	0.007	0.028	J12
	3.0339	4.9323	0.6151	0.009	0.035	H05+B06
V68	2.8092	4.5717	0.6145	0.005	0.028	D12
V70	2.0573	3.3115	0.6213	0.004	0.024	D12
V75	3.1839	5.1595	0.6171	0.006	0.023	D12
	3.1840	5.1506	0.6182	0.008	0.032	J12
V87	2.7973	4.5288	0.6177	0.006	0.034	D12
V97	2.9857	4.879	0.6147	0.005	0.026	D12
		4.868	0.6133	0.005	0.025	D12
		4.857	0.6119	0.005	0.024	D12
	2.9856	4.8597	0.6144	0.010	0.050	D12a
	2.9857	4.8703	0.6130	0.005	0.022	D12b
	2.9857	4.8370	0.6173	0.008	0.041	H05+B06
V125	2.8586	4.654	0.6142	0.006	0.028	D12
		4.640	0.6161	0.005	0.022	D12
		4.590	0.6229	0.005	0.025	D12
	2.8586	4.6343	0.6168	0.006	0.031	J12
	2.8587	4.6344	0.6168	0.009	0.047	H05+B06
V126	2.8702	4.604	0.6234	0.005	0.023	D12
V132	2.9424	4.7913	0.6141	0.011	0.053	D12
	2.9425	4.7497	0.6195	0.011	0.057	B06
V147	2.8863	4.6947	0.6148		0.028	D12
V208	2.9552	4.8058	0.6149	0.013	0.065	D12
V209	2.8712	4.6800	0.6135		0.033	D12
V210	2.8333	4.6185	0.6135		0.044	D12
V253	3.0066	4.8720	0.6171		0.079	D12
V259	2.9984	4.8773	0.6148		0.026	D12
V261	2.2485	3.6237	0.6205		0.069	D12

^aFourier amplitude in V band.

^bAmplitude ratio in V band. Italics denote B -band amplitude ratio of non-calibrated data.

^cD12a and D12b denotes first and second parts of the D12 data.

a strong bump preceding maximum light in the large-amplitude phase of its modulation, which feature is typical in RRc stars, thus its 2O classification is rejected. Second-overtone status of two more stars with no color information (V188 and V292) is also proposed based on their short periods and small-amplitude smooth light curves.

4. ADDITIONAL FREQUENCIES AT 0.61 FREQUENCY RATIO

The $f_{0.61}$ frequency ($f_{0.61} \approx f_{1O}/0.61$) is detected in 14 RRc and in four double-mode stars in M3. Thus, the phenomenon is observed in 38 and 40% of RRc and RRd stars, respectively. Fig. 5 and Fig. 6 show the spectral windows, the amplitude spectra and the prewhitened spectra of V75 and V97, as examples for the detection of the $f_{0.61}$ frequency in the D12 data.

Table 5 lists the RR_{0.61} stars and gives their frequencies, frequency ratios, $A_{0.61}$ amplitudes in V band and the $A_{0.61}/A_{1O}$ amplitude ratios. The previous CCD data (C01, K98, H05, B06, and J12) of these variables have also been re-investigated and the $f_{0.61}$ frequency was de-

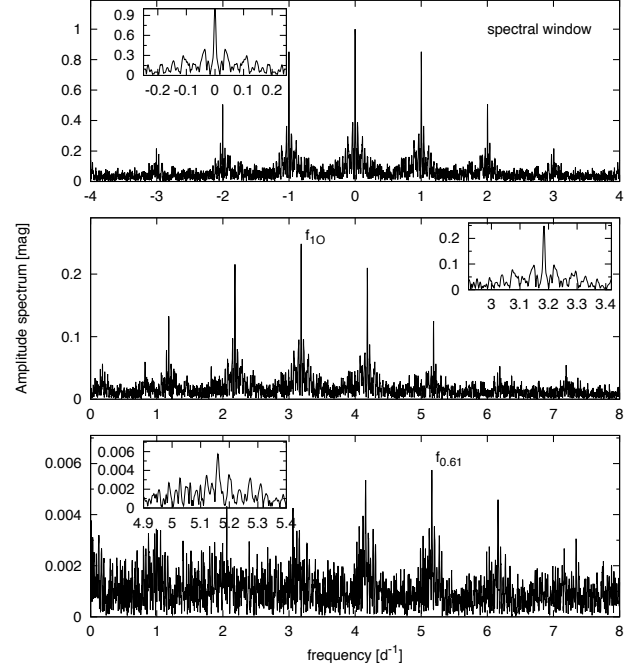


FIG. 5.— Spectral window (top), amplitude spectrum, and spectrum of the data prewhitened for the overtone-mode frequencies of V75 in the V band. The insets enlarge the main peaks. The detection of the $f_{0.61}$ frequency in the prewhitened spectrum of V13 is shown in Section 5.

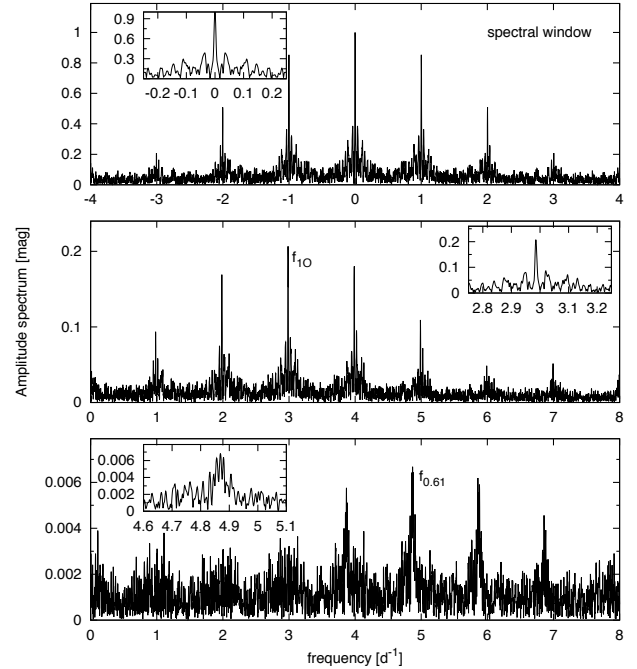


FIG. 6.— The same spectra as in Fig. 5 for V97. The $f_{0.61}$ frequency is a complex multiplet in this case.

tected in six stars in the archive photometric data; these results are also given in Table 5.

4.1. Stability of the $f_{0.61}$ frequency

The $f_{0.61}$ frequencies detected in the archive CCD and D12 data differ from each other in most of the cases

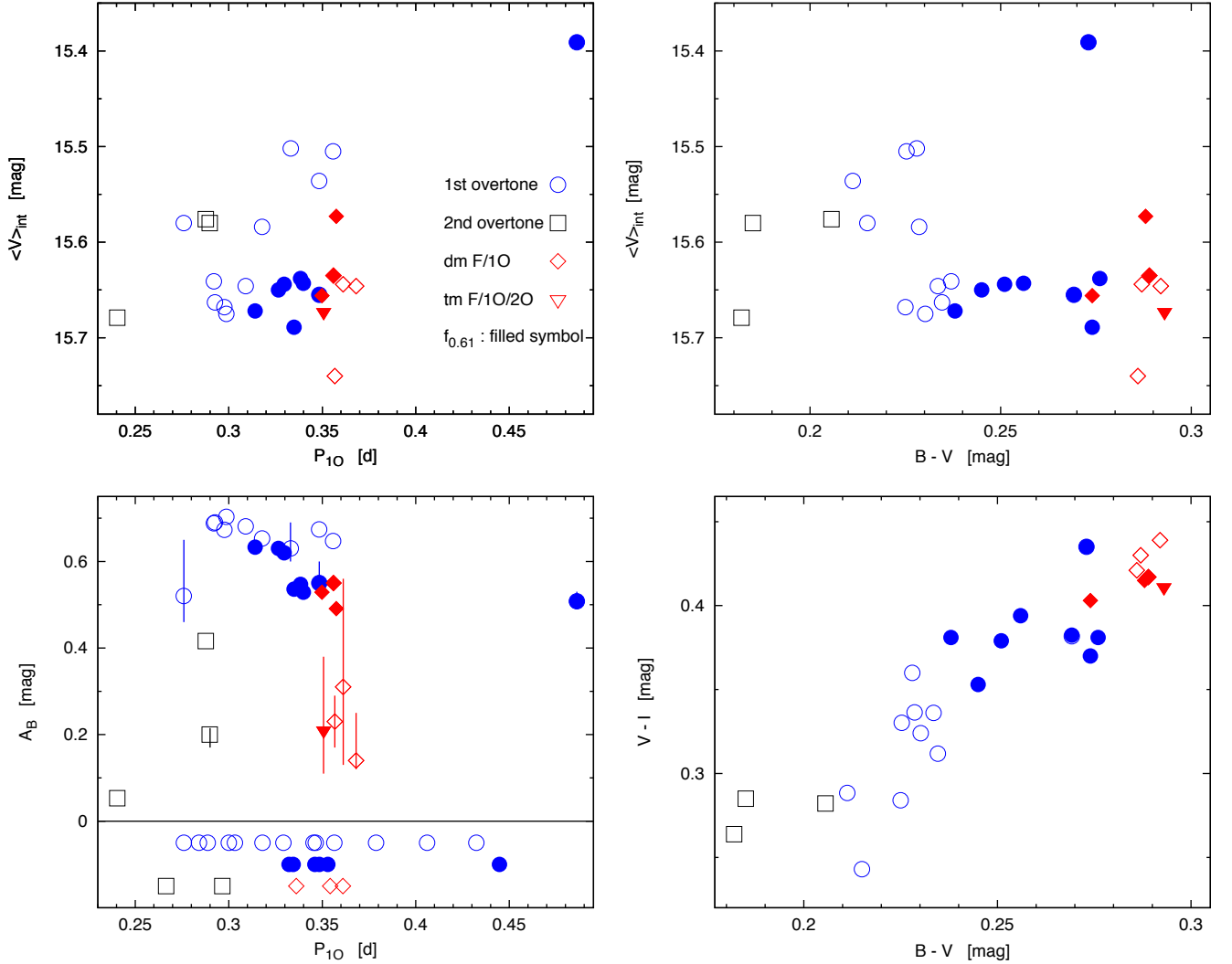


FIG. 7.— The same as in Fig. 3. $RR_{0.61}$ stars are shown by filled symbols.

and the differences are too large to be explained by the $\sim 0.0001 - 0.001 \text{ d}^{-1}$ uncertainty of the frequencies.

The $f_{0.61}$ frequency is a discrete, stable signal in V75, but this component has a complex, multiple structure in V97, indicating probable amplitude and/or phase changes of the signal. A complex structure of the $f_{0.61}$ frequency is detected in the residual spectra of four variables in the D12 data. The stability of the frequency solution of these stars is checked on V97. The first and second parts of the observations are analyzed separately, and these results are also given in Table 5. The frequency and the amplitude of the $f_{0.61}$ component are significantly different in the two parts of the data. This, and the differences between the results of the D12 and the archive data imply that, instead of real multiplets, amplitude and/or frequency changes of the $f_{0.61}$ component should be behind its complex appearance. This is in contrast with the stability of the radial-mode frequency of $RR_{0.61}$ stars. While as large as $\sim 1\%$ differences are detected in the frequency values of the $f_{0.61}$ component,

the variation in the frequency of the radial-mode is lower than 0.01% (see Table 5).

4.2. On the light curves of $RR_{0.61}$ stars

The appearance of the $f_{0.61}$ frequency in different groups of stars are indicated in Figs. 1 and 2. The light curves of these variables are more sinusoidal than the light curves of overtone variables without any additional feature. While a pronounced bump characterizes the light variation of normal RRC stars just preceding light maximum, the bump in these stars is marginal if it exists at all. The light curves of the 1O mode of double mode stars have a similar shape as $RR_{0.61}$ stars have, irrespective of whether or not the $f_{0.61}$ frequency is detected in their spectra (see also Fig. 20 in Section 7).

According to the best-quality light curves shown in Fig. 1, there is no star contradicting this scheme. Each $RR_{0.61}$ star displays a sinusoidal light curve with a strongly reduced bump, similar to the overtone-mode light curve of double-mode stars, while the light curves of normal RRC stars are asymmetric and their bump is

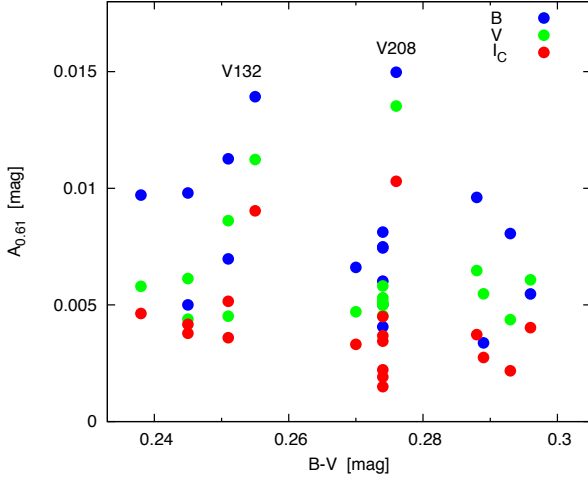


FIG. 8.— B -, V -, I_C -band amplitudes of the $f_{0.61}$ frequency component vs $B-V$. No amplitude decrease of the $f_{0.61}$ frequency towards smaller color indices is detected. The two largest $f_{0.61}$ -amplitude stars are labeled.

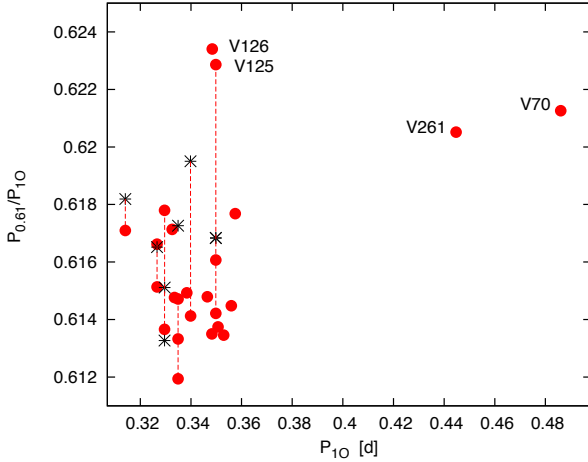


FIG. 9.— Petersen diagram, period ratio vs P_{10} of variables showing the $f_{0.61}$ frequency component. The detected frequencies of the same star are connected with dashed lines (see data in Table 5). Results for archive CCD data are shown by asterisk.

pronounced. The only exception is V70, which is a highly evolved, over-luminous, long-period RRc star.

The non-calibrated light curves of variables affected by crowding shown in Fig. 2 follow the same pattern with some exceptions. Each $RR_{0.61}$ star of this sample fits to this scheme, their bumps are marginal. However, similar, sinusoidal light curves are found among normal RRc stars, as well. Because of the larger noise level, the detection limit of the additional frequency is larger in these cases than in the best-quality light curves. Therefore, it cannot be ruled out that these ‘outliers’ may, in fact, exhibit the ‘0.61’ feature as well, but the quality of the light curve does not allow its detection. There is an evolved, long-period star in this sample (V170), too, which shows the $f_{0.61}$ frequency.

The Fourier parameters of the light curves (Fig. 4) also reflect the above described diversity. While mono-periodic RRc stars have detectable amplitudes up to ten or even larger harmonic orders in V band, we can find

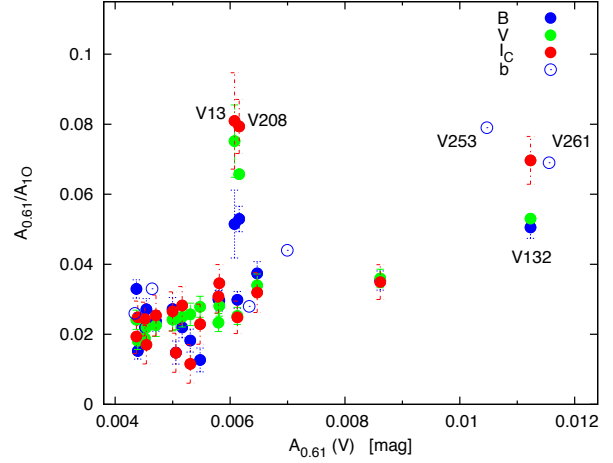


FIG. 10.— B -, V -, I_C -band and non-calibrated B -filter amplitude ratios ($A_{0.61}/A_{10}$) vs the Fourier V amplitude of the $f_{0.61}$ frequency (B -filter amplitude of the non-calibrated data). For multiple $f_{0.61}$ signals each component is shown.

harmonics only up to 6 – 8 with amplitudes above the noise limit in stars showing the $f_{0.61}$ frequency. The Fourier amplitudes and amplitude ratios of the $RR_{0.61}$ stars are systematically smaller than that ones of normal RRc stars. Only double mode, Blazhko, and 2O stars have amplitudes and amplitude ratios as low as $RR_{0.61}$ stars have over the third harmonic order. The phase differences of the $RR_{0.61}$ stars do not show any peculiar behavior, they follow the trend defined by other RRc stars.

4.3. Parameters of $RR_{0.61}$ stars

The location of $RR_{0.61}$ stars is also special in the panels shown in Fig. 7. These plots are the same as in Fig. 3, but the $RR_{0.61}$ stars are marked by filled symbols for clarity. The $f_{0.61}$ frequency component is not detected in any of the stars with period shorter than 0.31 d, while it appears in the residual spectrum of each, not over-luminous RRc star at longer periods, and in four double-mode stars. None of normal-period, bright, evolved RRc stars shows the $f_{0.61}$ frequency, but it is detected in two of the five, highly evolved stars. The periods of these 10 pulsators (V70, $P = 0.486$ d and V261, $P = 0.445$ d) are longer than the 10 period of double-mode stars; they are in the period range of the shortest-period fundamental-mode variables. Although the magnitude calibration of V261 is uncertain, as its period and light-curve shape is similar to those of V70, the brightest, longest-period RRc star in M3, it is very probable that there are no significant differences between the luminosities and temperatures of V261 and V70.

The separation of RRc and $RR_{0.61}$ stars is the most pronounced in the $B-V-V-I$ panel of Fig. 7. It seems that the $f_{0.61}$ phenomenon occurs only in the hotter half of the temperature range of overtone and double-mode stars in M3. There are only three double-mode stars (V44, V99, and V166) in the hotter half of the complete RRc/RRd sample where the $f_{0.61}$ frequency is not detected. However, these stars exhibit complex Blazhko modulation of both modes and the mean amplitude of their 10 mode is only about the half of the amplitude of the other RRc stars and of RRd stars with dominant

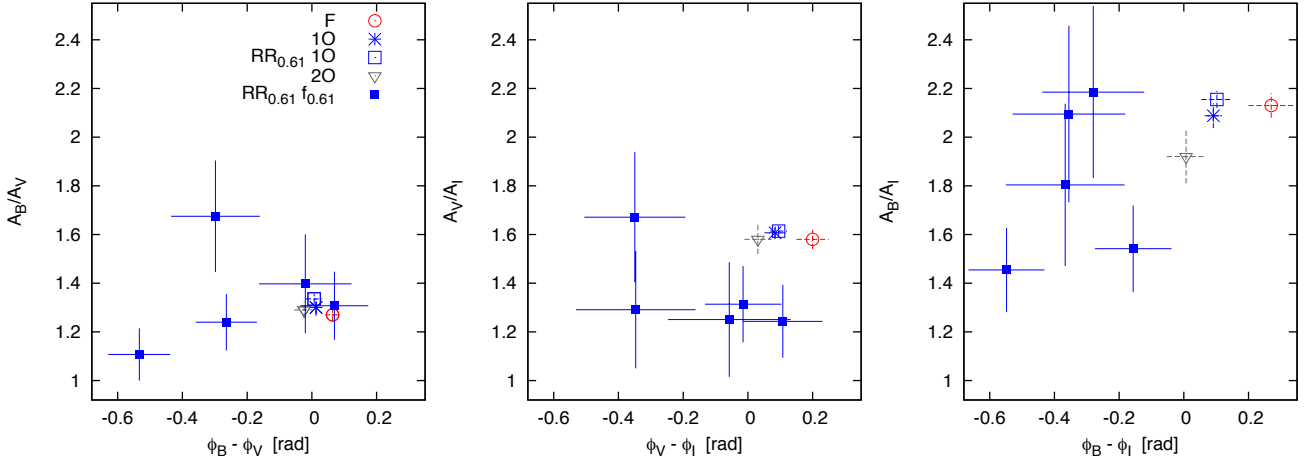


FIG. 11.— The amplitude ratios and phase differences of single-mode F, 1O, and 2O variables, the first-overtone mode of $RR_{0.61}$ stars, and the $f_{0.61}$ frequency component in the B , V and I_C bands are plotted. With the exception of the $f_{0.61}$ components, the mean values of the parameters and the standard deviations of the samples are shown. Because of their small amplitudes, the amplitude ratios and phase differences of the $f_{0.61}$ frequency components have large errors, therefore, results only of the 5 largest S/N components are shown. The formal errors, which, however, may underestimate the true uncertainties of the $f_{0.61}$ components, are indicated.

overtone mode. Supposing that the $A_{1O}/A_{0.61}$ amplitude ratio is not extremely large (as in the case of V13, the only RRd star with dominant fundamental mode showing the $f_{0.61}$ component), the estimated amplitude of the $f_{0.61}$ frequency might be below the detection limit in these three RRd stars. The $f_{0.61}$ frequency is not detected in the RRd stars with uncalibrated, noisy light curves either, but the $f_{0.61}$ frequency may be hidden in the noise in these stars, too. Therefore, we cannot exclude the possibility that the $f_{0.61}$ frequency is a common property of RRd stars and of the hotter half of the RRC sample. In contrast, none of the RRC stars with $B - V \lesssim 0.24$ and $V - I \lesssim 0.35$ mag shows the $f_{0.61}$ signal.

Fig. 8 shows the amplitudes of the $f_{0.61}$ frequency components plotted against the $B - V$ color index. No systematic amplitude decrease of the $f_{0.61}$ frequency is detected with decreasing color index. Consequently, the lack of $RR_{0.61}$ stars at hotter temperatures as shown in the $B - V$ vs $V - I$ panel of Fig. 7 cannot be explained by a systematic reduction of the amplitude of the $f_{0.61}$ component towards hotter temperatures.

Fig. 9 plots the observed $P_{0.61}/P_{1O}$ period ratios of the 18 $RR_{0.61}$ stars. Each component of the stars with multiple $f_{0.61}$ frequencies and results for the archive CCD data are shown. The period ratios fall in a very narrow range between 0.612 and 0.623. The four largest values are detected in peculiar stars. Two of them, V70 and V261 are evolved, extremely long-period variables. The additional frequency is a strongly varying signal with three $f_{0.61}$ components appearing in the whole data set of V125. The period ratios of two of these components and those obtained from the H05, B06 and J12 data fall in the 0.614 – 0.617 range. The largest period ratio is detected in V126. The $f_{0.61}$ component is one of the weakest in this star, thus the frequency solution is somewhat uncertain in this case.

Fig. 10 shows the $A_{0.61}/A_{1O}$ amplitude ratios in B , V , and I_C bands (and for the non-calibrated B -filter data)

derived from the D12 data. Most of the amplitude ratios are in the 0.02 – 0.04 range. The amplitude ratios of five stars (V13, V132, V208, V253, and V261) are even larger, they are about 0.05 – 0.08. The increased $A_{0.61}/A_{1O}$ amplitude ratios are the consequences of either the relatively large amplitude of the $f_{0.61}$ component or the reduced amplitude of the overtone mode as in the case of V13. No general, systematic difference between the amplitude ratios in the B , V , I_C bands is seen for most of the $f_{0.61}$ components. However, the three large amplitude-ratio stars with BVI photometric data (V13, V132, and V208) show an anomalous tendency, their $A_{0.61}/A_{1O}$ amplitude ratio is the largest in the I_C and the smallest in the B band. However, taking into account that the formal errors may underestimate the true uncertainties seriously, this result has to be taken with caution.

The amplitude ratios and phase differences of single-mode F, 1O, 2O variables, and the first-overtone mode of $RR_{0.61}$ stars (mean values and the standard deviations of the samples), and of the $f_{0.61}$ frequency component are plotted in Fig. 11 for the B , V , and I_C bands. The amplitude ratios and phase differences of the $f_{0.61}$ components have large errors because of their small amplitudes. Therefore, results are shown only for the five best, smallest-error components. There is no detectable difference between the positions of the overtone mode of the mono-periodic and the $RR_{0.61}$ stars shown by an asterisk and a square, respectively. The phase differences of the F and 2O modes are at slightly larger and smaller values than the phase differences of the 1O. The amplitude ratios and phase differences of the best S/N-ratio $f_{0.61}$ components form a separate group in the V, I and B, I planes with no overlap with the regions of the radial modes. Although the errors even of the best signals are significant, the homogeneous location of these components implies that the amplitude ratios and phase differences of the $f_{0.61}$ frequency and of the radial modes may indeed be different. However, more accurate multicolor

observations of RR_{0.61} stars are needed to locate the positions of the $f_{0.61}$ component in the amplitude-ratio and phase-difference planes undoubtedly.

4.4. Comparison with other results

The $f_{0.61}$ frequency has been detected in several overtone RR Lyrae stars previously. A complete list of these objects was given by Moskalik et al. (2015) recently. Moreover, Netzel et al. (2015) identified 147 Galactic bulge objects in the OGLE-III photometry which all showed the peculiar frequency component in the 0.60 – 0.65 period-ratio range. The period ratios of the OGLE sample defined two sequences, one at 0.613 and a less populous group at 0.631 ratio. Moskalik et al. (2015) argued that this frequency cannot be identified with any radial mode. They have concluded, that most probably the $f_{0.61}$ frequency corresponds to a strongly trapped, unstable, high-degree ($l \geq 6$) non-radial mode.

Color information may help mode identification of non-radial modes (see e.g. Garrido 2004), however, RR Lyrae stars still lack appropriate modeling to exploit this possibility.

Similar period ratios were detected in numerous Magellanic Cloud 10 Cepheids (Soszyński et al. 2008; Moskalik & Kołaczowski 2008, 2009; Soszyński et al. 2010). The period ratios of these stars fell on three parallel lines, separated by about 0.02 in period ratios in the $P/P_{10} - \log P_{10}$ Petersen diagram. In a theoretical work, Dziembowski (2012) concluded that in Cepheids ‘the only unstable modes that may reproduce the observed period ratio are f -modes of high angular degrees ($l = 42 - 50$)’. This hypothesis predicts very large ($35 - 70 \text{ km s}^{-1}$) broadening of the spectral lines of these stars, which has not been verified observationally yet.

RR Lyrae stars are known to be slow rotators (Peterson, Rood & Crocker 1995; Preston & Chadić 2013) with microturbulent velocity of about $2 - 3 \text{ km s}^{-1}$ (Govea et al. 2014, and references therein) and no significant line broadening of any RR Lyrae stars has been detected previously.

Spectroscopic data of M3 variables were also collected by Hectochelle@MMT (Szentgyörgyi et al. 2011) with ~ 20000 spectral resolution in the 5150 – 5300 Å wavelength range parallel with the photometric observations in 2012. The details of the observations and data reduction process will be published in a separate paper.

Six overtone RRL stars belonged to the targets of the spectroscopic observations. Three of them, the RRc stars V56, V87 and the RRd star, V97 show the $f_{0.61}$ frequency. The others are normal RRc stars (V86 and V107), and one of them, V140 shows strong Blazhko modulation. We have checked the line broadenings of these stars in pulsation phase near to minimum brightness. Fig 12 shows the line profiles of the Mgb (5183.604 Å) absorption line, the deepest, non-blended line in the spectra of the 6 RRc/RRd stars and the best fitting models using the alpha enhanced synthetic spectra ($Z = -1.5$) of the Asiago stellar library (Munari et al 2005). The atmospheric parameters (T_{eff} , $\log g$) were estimated from the observed colors assuming $E(B - V) = 0.01$ mag interstellar reddening using the synthetic colors published in Castelli & Kurucz (2003). Uncertainties of the

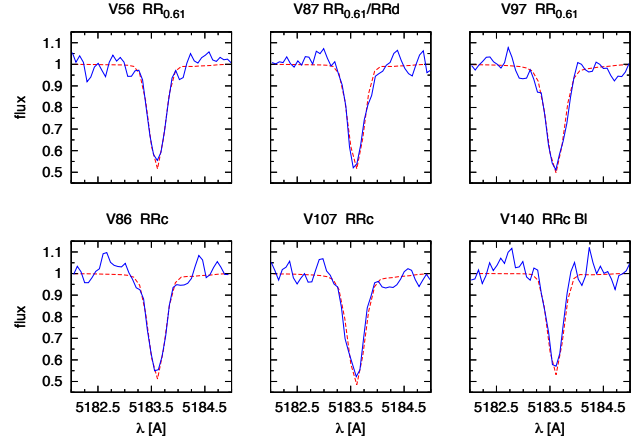


FIG. 12.— The Mgb (5183.604 Å) absorption line in the spectra of 6 RRc/RRd stars at the pulsation phase around minimum brightness. Stars showing the $f_{0.61}$ frequency are in the top panels. The line profiles are fitted with synthetic spectra of appropriate physical parameters and of rotational velocities of 5 km s^{-1} ($v \sin i$) for each star. No systematic difference is detected between the broadenings of the line profiles of RR_{0.61} and other 10 stars.

physical properties account for about $1 - 2 \text{ km s}^{-1}$ errors of the results.

The broadenings of the line profiles are estimated to be about 5 km s^{-1} for all the six stars (Fig. 12). The line broadening of the three RR_{0.61} stars are thus the same as for RRc stars not showing the $f_{0.61}$ component. The lack of any systematic difference between the broadenings of the line profiles of RR_{0.61} and other 10 RRL stars in M3 contradicts the high-order f -mode explanation of the $f_{0.61}$ frequency proposed by Dziembowski (2012).

Amplitude and phase variations of the $f_{0.61}$ component on a time scale of 10 – 200 d was detected in the Kepler data (Moskalik et al. 2015). However, the Kepler data revealed very low-amplitude (tenths of mmag) variations of the radial mode on similar timescale, as well. Such variations are below the detection limit of the D12 data.

The light curves of the Kepler RRc stars with the $f_{0.61}$ frequency component are also sinusoidal or show only a marginal bump (see fig. 2 in Moskalik et al. 2015), as in M3. Although no H emission in the pre-maximum phase of RRc stars has been reported, the pre-maximum bump in RRab stars is related to a strong atmospheric shock (Gillet & Fokin 2014, and references therein). The lack of a pronounced bump on the light curves of RR_{0.61} stars might indicate that the formation of atmospheric shocks and the excitation of the $f_{0.61}$ frequency are mutually exclusive.

The amplitude ratio of the $f_{0.61}$ frequency relative to the amplitude of the radial mode is in the 0.02 – 0.08 range in M3. The amplitude ratios are in a similar range in all the other samples of stars showing the $f_{0.61}$ frequency. They are 0.01 – 0.04, 0.02 – 0.05, and 0.02 – 0.08 in Cepheids (Soszyński et al. 2010), in the RRc stars observed by the Kepler space telescope (Moskalik et al. 2015), and in the Galactic bulge (Netzel et al. 2015), respectively. The $f_{0.61}$ frequency is thus a relatively small-amplitude signal according to each of these studies.

Each of the four RRc stars observed by the Kepler mission shows the $f_{0.61}$ frequency, moreover, period doubling of this component are detected in these stars

(Moskalik et al. 2015). Half-integer frequency of the $f_{0.61}$ component appears in four of the 147 Galactic bulge RR_{0.61} stars, too (Netzel et al. 2015). The amplitudes of these half-integer frequencies are smaller than 0.003 mag with one exception, the amplitude of the $f_{0.61}/2$ frequency of KIC 5520878 is 0.006 mag. No $kf_{0.61}/2$ half-integer frequency is detected in M3, however, signals with amplitude smaller than 0.004 mag are below the detection limit of the D12 data.

The $f_{0.61}$ frequency is detected in 38% of the RRc/RRd stars in M3. Moskalik et al. (2015) found that almost every RRc and RRd star observed from space showed the $f_{0.61}$ frequency, consequently the phenomenon must be very common. Concerning Cepheids, $\sim 9\%$ of the 10 pulsators showed the $f_{0.61}$ frequency in the SMC (Soszyński et al. 2010), but none of the shortest period ones ($\log P_{10} < 0.056$) as noticed by Dziembowski (2012). It seems that in this sample there is a tendency that the phenomenon does not occur in the hottest stars.

About 20% of the RR_{0.61} stars are bona-fide double-mode pulsators in M3. There are 4 and 2 RRd stars in the samples of 23 field and globular cluster (Moskalik et al. 2015), and 147 Galactic bulge RR_{0.61} stars (Netzel et al. 2015), respectively. Thus the frequency of double-mode pulsators in these data sets are 17, and 1.4%. The very low rate of RRd stars showing the $f_{0.61}$ frequency in the Galactic bulge is the consequence of the fact that the occurrence rate of RRd stars among overtone variables in the Galactic bulge (Soszyński et al. 2011) is only $\sim 2\%$.

The homogeneity of GC data allows to determine the positions of RR_{0.61} stars relative to other overtone variables using their measured parameters. Each RRc star close to the blue edge of the double-mode region – based on their periods and colors – and four RRd stars show the $f_{0.61}$ frequency. It is also detected in some highly evolved, bright, long-period variables, which temperature (indicated by their colors) fall in the temperature range covered by the other RR_{0.61} stars. The similarity of the light-curve shapes of RR_{0.61} and RRd stars (with reduced or missing bump) and their physical parameters (based on the colors and periods) indicates that the $f_{0.61}$ phenomenon might be strongly connected to the double-mode state. Maybe, the complex dynamics of the observed and/or hidden double-mode pulsation is behind the excitation of this peculiar frequency component.

5. PERIOD DOUBLING

A new feature of the pulsation of RRab stars, the alternating shapes of their light curves (i.e., period doubling) was recognized, thanks to the high-precision time-resolved photometry of the Kepler space-telescope data (as a review see Szabó 2014). Half-integer frequencies of the fundamental-mode frequency are the signature of the phenomenon in the Fourier spectrum. The observed half-integer frequencies had the highest amplitude typically at $3f_F/2$ and they were detected exclusively in stars showing Blazhko modulation in the Kepler data. Moskalik et al. (2015) reported that half-integer frequencies also appear in RRc stars, however, in this case the period doubling was connected to the additional, $f_{0.61}$ frequency, instead of the radial-mode pulsation component. Period doubling of the third (third radial overtone or non-radial) mode of OGLE-BLG-RRLYR-24137 was also reported by Smolec (2015b). Using turbulent, con-

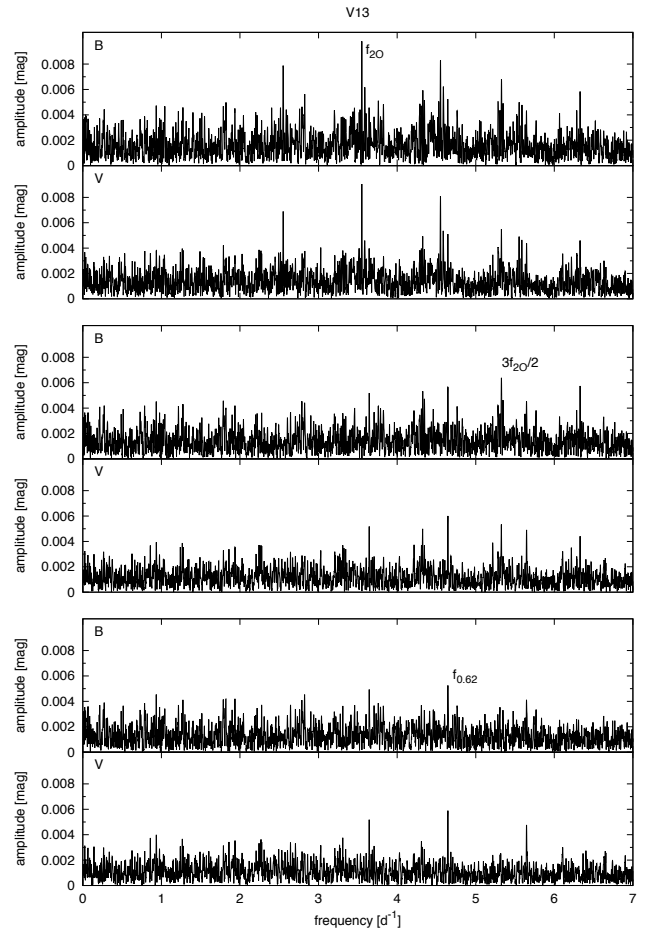


FIG. 13.— *B* and *V* residual spectra of V13 in consecutive prewhitening steps. The largest signal appears at the f_{20} (2O-mode) frequency in the data prewhitened for the f_F and f_{10} frequency components, their modulations and the linear-combination terms (top panels). In the next prewhitening steps, frequencies are detected at $3f_{20}/2$, and then, at $f_{0.61}$ (middle and bottom panes).

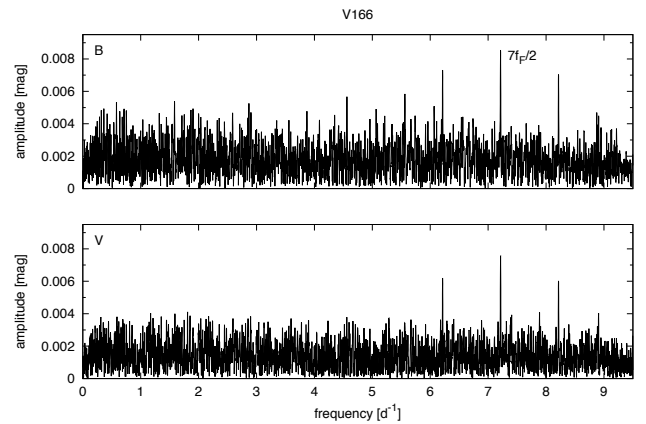


FIG. 14.— *B* and *V* residual spectra of V166, prewhitened for the f_F and f_{10} frequency components and their modulations and linear-combinations, are drawn. Frequency appears at the position of the half-integer frequency, $7f_F/2$.

TABLE 6
PERIOD DOUBLING DETECTED IN THE M3 DATA.

Star	Mode ^a	k^b	P [d]	Error	$P_{PD} - 2P/k$	$A(V)$
V13	2O		0.28160	0.00003		0.010
		3	0.18775	0.00003	0.00002	0.005
V166	F		0.48504	0.00001		0.200
		7	0.13861	0.00002	0.00003	0.008

^aPulsation mode period-doubling belongs to.

^bOrder (k) of the half-integer frequency, $2P/k$.

vective hydrodynamical models and amplitude equation formalism, Kolláth et al. (2011) and Buchler & Kolláth (2011) argued that a 9:2 resonance between the fundamental and the ninth radial overtone (which is a strange mode) is responsible for the observed period doubling of RRab stars and it might be taken part in the generation of the Blazhko modulation. In contrast to this claim, citing Moskalik et al. (2015) ‘it is unclear which (if any) resonance is responsible for the period doubling in the RRC and RRd variables.’

The D12 data revealed period doubling of two multi-mode RR Lyrae stars in M3, as well. It is connected to the second overtone of the triple-mode star, V13, and to the fundamental mode of the double-mode star, V166. Both stars show Blazhko modulation of the fundamental and the overtone modes. Table 6 gives the main properties of the frequencies connected to period doubling of these stars. These frequencies appear at the exact value of the half-integer frequencies within the uncertainty limits (3σ).

The B and V residual spectra, prewhitened for the f_F and f_{10} frequency components, their modulations and the linear-combination terms, are shown for V13 and V166 in Fig. 13 and Fig. 14, respectively. The frequency of the 2O appears first in V13, then, in further prewhitening steps, frequencies are detected at $3f_{2O}/2$, and finally, at $f_{0.61}$. The prewhitened spectra of V166 show a signal at the $7f_F/2$ half-integer frequency with a high significance.

Despite the fact that the amplitude of the 2O pulsation mode is only 0.010 mag, quite surprising that the half-integer frequency is connected to the 2O mode and not to the most strongly modulated large-amplitude fundamental mode. The amplitude ratio of the half-integer frequency to the 2O is quite large, $A_{3f_{2O}/2}/A_{f_{2O}} = 0.50$. We note that larger amplitude ratio of any half-integer component was detected only in some of the $f_{0.61}$ frequencies reported by Moskalik et al. (2015) and Netzel et al. (2015). The half-integer frequency was connected also to a small-amplitude frequency component and not to the large-amplitude radial mode in these cases.

Looking for a candidate resonance behind the period doubling of the 2O mode of V13, a low-order one, $3f_{2O} = 2f_{4O}$, may be valid in the 6800 – 6900 K temperature range based on the period-ratio grid of radial modes (fig. 3 in Kolláth et al. 2011). According to the temperature calibration of Cacciari et al. (2005), double-mode stars fall exactly into this temperature range in M3. However, as the fourth overtone is strongly damped (see fig. 4 in Kolláth et al. 2011), its excitation, even in resonance position is dubious. Why period doubling of the small-amplitude 2O appears in V13 and whether its $3f_{2O}/2$

frequency can be explained by a resonance with some high-order overtone remain a question to be answered. Besides the period doubling feature observed in RRab stars what V166 seems to be following, the behavior of V13 is pointing to some additional mechanism also taking part in this effect.

Period doubling of the overtone mode of single or double-mode stars or the $f_{0.61}$ frequency component is not detected in M3.

6. BLAZHKO EFFECT

Equally spaced asymmetric triplets, multiplets, which are characteristics for Blazhko RRab stars, are detected in the residual spectra of seven overtone and double-mode stars. A small-amplitude component near the radial-mode frequency appears in the residual spectra of two further stars (V126, V203). Although the non-radial mode origin of these latter signals cannot be excluded, we classify these stars also as Blazhko variables. Six of the Blazhko stars are multiperiodic besides their light-curve modulation. V13, V44, V99, and V166 are double(triple)-mode stars, and the $f_{0.61}$ frequency component is detected in V70, and V126. Some specialties of the Blazhko properties of overtone stars in M3 were already discussed in Jurcsik et al. (2014). The Blazhko stars are denoted by thick symbols in Figs. 3 and 4.

Table 7 lists the pulsation and modulation periods, the amplitudes of the amplitude- and phase-modulations, and data and comments on the main features of the modulation detected in the D12 of Blazhko stars.

The light curves of the strongly modulated stars in eight phases of the Blazhko cycle are shown in Fig. 15. The data of multi-mode pulsators are prewhitened for the signals of the pulsation and the modulation of the other radial mode, the $f_{0.61}$ frequency, and for the linear-combination terms. Note that the modulation periods of the two modes are different in V44, V99, and V166.

The light curves in Fig. 15, and the data given in Table 7 show that the modulation of RRC and the overtone-mode of RRd stars are phase modulations predominantly. The phase modulation of the overtone mode of V44 and V99 is larger than one third of the pulsation period. This is in contrast with the Blazhko behavior of the fundamental-mode of RRd and RRab stars. Their modulation is dominated mostly by amplitude variation or their amplitude and phase modulations have similar strength.

Pre-maximum bump appears only in the two single-mode Blazhko stars (V140 and V168) in some phases of their Blazhko cycle. Both of these stars are at high luminosity but at different temperatures as indicated by their magnitudes, periods, and color indices (see Fig. 3). The bump is observed at the opposite phases of the modulation of the two stars. The shock-induced phenomenon appears in V140 and V168 when the phase-shift is negative (period is decreasing) and the amplitude is small and when the phase-shift of the light curve is positive (period is increasing) and the amplitude is large. The temperature difference between the two stars may explain this reverse behavior. The temporal appearance of the bump in Blazhko RRC stars implies that their main physical properties vary during the Blazhko cycle, similarly as detected in Blazhko RRab stars (see e.g. Jurcsik et al. 2008; Sódor et al. 2011; Jurcsik et al. 2012b).

TABLE 7
BLAZHKO PROPERTIES ACCORDING TO THE D12 DATA.

Star	P_{puls} d	$P_{\text{mod}}^{\text{a}}$ d	f_{mod} d^{-1}	$A_{A_{\text{mod}}}^{\text{b}}$ mag	$A_{\text{ph}_{\text{mod}}}^{\text{c}}$ rad	Summary of the main modulation components
V13	0.35072	139(1)	0.007	0.20	1.25	$-f_{\text{m}}, -2f_{\text{m}}$ at the first and 2nd harmonic orders
	0.47949	139(1)	0.007	0.42	0.85	dominant $-f_{\text{m}}$, some small amplitude $+f_{\text{m}}, -2f_{\text{m}}$ in the first 4 harmonic orders
V44	0.36812	56.0(8)	0.018	0.15	2.20	complex multiplets at the 1st and 2nd harmonic orders
	0.50377	97.0(2)	0.010	0.45	1.00	complex modulation with strong $-f_{\text{m}}, -2f_{\text{m}}$ and $\pm f_{\text{m}}/2$ components
V70	0.48607	150(10)	0.007	0.01	0.10	triplet with 0.007 and 0.006 mag amplitude of the $-f_{\text{m}}, +f_{\text{m}}$ components
V99	0.3611	40.3(2)	0.025	0.35	2.20	$+f_{\text{m}}, +2f_{\text{m}}, +3f_{\text{m}}$ components at the 1st and 2nd harmonic orders
	0.4821	≈ 450	≈ 0.002	> 0.25	> 0.2	triplet with 0.050 and 0.025 mag amplitude of the $-f_{\text{m}}$ and $+f_{\text{m}}$ components
V126	0.34841	15.0(2)	0.067	0.03	0.04	one component at $f + f_{\text{m}}$ with 0.006 mag amplitude
V140	0.33316	28.8(1)	0.035	0.10	0.63	triplets at least up to the 5th harmonic order
V166	0.35672	43.5(5)	0.023	0.08	0.50	one component at $f - f_{\text{m}}$ with 0.023 mag amplitude
	0.48504	71.5(3)	0.014	0.45	1.20	members of the quintuplets (dominant $-f_{\text{m}}, -2f_{\text{m}}$) up to the 4th harmonic order
V168	0.27594	23.0(1)	0.043	0.16	1.00	members of the quintuplets up to the 4th harmonic order
V203	0.28979	118(5)	0.008	0.03	0.15	one component at $f + f_{\text{m}}$ with 0.007 mag amplitude

^aThe error in the last digit is given in parenthesis.

^bFull amplitude of the amplitude modulation in V band.

^cFull amplitude of the phase modulation in V band.

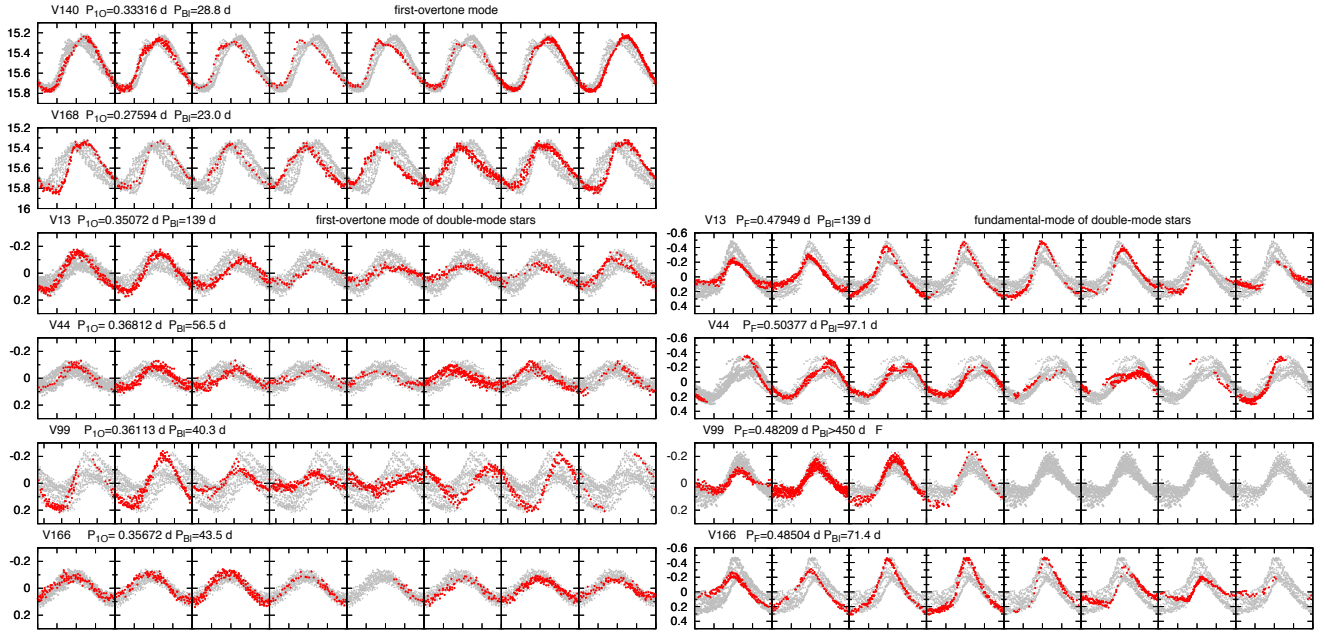


FIG. 15.— V light curves of the Blazhko stars in eight phases of their main modulation cycle are shown. The light curves of the double-mode stars are prewhitened for the light variation of the other mode and for the linear-combination terms.

6.1. Notes on the Blazhko properties of the stars

Besides the analysis of the D12 data, the archive photographic and CCD data collected in J12 have also been checked to follow the changes of the Blazhko properties on a long time scale. Further details on the pulsation/modulation of Blazhko RRd stars are given in Section 7.

V13 is a triple-mode star (F/1O/2O) with a dominant fundamental mode (Sect. 7.1). The F and 1O modes are modulated with the same period and the amplitude and phase variations of the modes are in anti-phase with each other (see also in Jurcsik et al. 2014), similarly to 1O/2O Blazhko Cepheids (Moskalik & Kołaczowski 2009). The amplitude ratios of the radial modes to the largest-amplitude modulation components are 2.3 and

1.6 for the F and 1O modes. Assuming that the pulsation/modulation amplitude ratios of each radial mode are similar, the modulation amplitudes are expected around 0.004 – 0.005 mag at the 2O frequency as the V amplitude of the 2O mode is 0.009 mag. However, no modulation component with amplitude larger than ~ 0.002 mag has been found; the largest amplitude signal close to the 2O frequency in the residual spectrum is the 1 d^{-1} alias component of the $f_{0.61}$ frequency (see Fig. 13). The previous CCD observations show a similar behavior as in the D12 data; anti-phase modulations of the fundamental and overtone modes with 144 d is derived from the combined K98, B06 and J12 data set.

V44 is a double-mode star with a dominant fundamental mode. The modulation of the fundamental mode is very complex; the main modulation period is 97 d but

modulations with the half and the twice of this period are also detected. The 56 d modulation of the overtone mode is dominantly phase modulation. Modulation of the fundamental mode with 52 d, 104 d, and 111 d are identified in the C01, H05+B06, and the J12 data, respectively. The photographic data show some signs of modulation with $P_m = 82$ d (JD 2420625 – 2424684), and 49 d (JD 2428963 – 2436991)

V70 is a very-long-period (0.486 d), over-luminous, evolved variable. The small-amplitude, 150-d modulation detected in the D12 data is dominantly phase modulation but amplitude changes are also observed mostly in the *B* band. The amplitude changes and the 150-d period of the modulation, which is shorter than the time span of the observations, make it unlikely that period change mimics Blazhko modulation in this case. The C01, H05, B06, and J12 data show significant residuals at f_{10} but with different separations. However, it is not clear in these cases whether these signals arise from strong period changes, which are typical in this star, or from light-curve modulation.

V99 is a double-mode star. The amplitude of the fundamental mode was increasing from 0.13 to 0.42 mag in the D12 data. The *V* amplitude of the fundamental mode was 0.20, 0.40 and 0.25 mag in 1992 – 1993, 1999 and 2009, respectively, according to the previous CCD observations (C01, H05, B06, and J12). The strong differences in the measured amplitudes of the fundamental mode indicates that the amplitude increase of the fundamental mode in the D12 data reflects Blazhko modulation on a time-scale of some hundred days and that the true amplitude of the fundamental mode can be even larger as measured in the D12 data. The overtone mode is strongly amplitude and phase modulated. Blazhko modulation of the overtone mode is identified in both the combined H05 and B06 and the J12 data with a period of around 40 d. **V126** is an RRc star showing a small-amplitude $f_{10} + f_m$ component. This RRc star is the longest period, not over-luminous one in the magnitude calibrated sample, which lies just at the blue edge of the double-mode region.

V140 is about 0.2 mag above the ZAHB. The J12 data show Blazhko modulation ($f_{10} + f_m$, $f_{10} - f_m$ and $2f_{10} - f_m$ components) with $f_m = 0.0689$ d⁻¹ ($P_m = 14.5$ d), which is half of the recent modulation period. The combined H05 and B06 observations yield similar light-curve modulation as observed in the D12 data. The pulsation and modulation periods were 0.333146 d and 29.0 d in 1999. The modulation period cannot be determined in the C01 data, but from the large residuals around the pulsation frequency we suppose that the light curve was modulated between JD 2448755 and 2449091 also. Similar modulation as observed in the CCD data is suspected in two parts of the photographic data; the $f_{10} + 0.068$ d⁻¹ (~ 15 d) component might be present in the data between JD 2422729 – 2424684 and JD 2433390 – 2435933, in the latter data set the $f_{10} - 0.034$ d⁻¹ (~ 29 d) component has a significant amplitude, too. The dominant modulation of V140 seems to switch between f_m and $2f_m$ time to time.

V166 is a double-mode star with a dominant fundamental mode. The fundamental mode shows strong amplitude and phase modulations, while the modulation of the

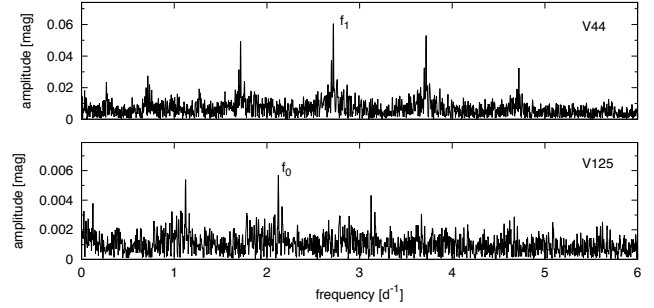


FIG. 16.— Residual spectra of the detected new radial-mode components of V44 and V125. The data are prewhitened for all the other signals.

overtone mode is modest. A 71 d modulation of the fundamental mode is also detected in the B06 and J12 data, and the large residual at f_F points to modulation of the fundamental mode in the C01 data, too.

V168 is a short-period, over-luminous star. The combined H05 and B06 data yield similar light-curve modulation as observed in the D12 data. The pulsation and modulation periods were 0.27595 d and 22.9 d in 1999. Modulation of the fundamental mode is also evident in the C01 data.

V203 is a second-overtone over-luminous variable. A frequency component at $f_{20} + 0.0096$ d⁻¹ is also detected in the J12 data with 0.008 mag amplitude.

7. DOUBLE- AND TRIPLE-MODE STARS

Nine double-mode stars have been identified in M3 previously. A thorough investigation of M3 RRd stars was given by (Clementini et al. 2004, hereafter C04). Double-mode nature of two more stars, V44 and V125, and the triple-mode behavior of V13 are detected in the D12 data. The residual spectra prewhitened for all the other signals in Fig. 16 depict the new radial modes of V44 and V125 in the D12 data like the 2O mode of V13 in Fig. 13.

The D12 data makes it possible to reveal the complex behavior of RRd stars. To inspect the changes of pulsation properties all the previous photometric data listed in Section 2 are also checked. The 200-d length of the D12 data allows to determine the pulsation frequencies of double-mode stars typically with a precision of the order of $10^{-4} - 10^{-5}$ d⁻¹. The frequency spectra of the previous photometric data are strongly biased; especially the 1 yr⁻¹ alias frequencies result in ± 0.003 d⁻¹ ambiguities of the solutions. Therefore, the frequencies published in the literature (K98, C01, C04, H05, B06, and J12) were revised in the sense that from the equally possible frequency solutions that was accepted which gave closest resemblance to the solution of the D12 data.

The main photometric properties of the double/triple mode stars obtained from the D12 data are given in Table 4. Table 8 summarizes the detected pulsation and modulation (discussed in Section 6) properties of double-mode stars at different epochs.

7.1. The triple mode star, V13

V13 The new frequency components detected in the residual spectra of V13 are shown in Fig 13. The frequency appearing at $f_F/f_x = 0.5873$ ($f_{10}/f_x = 0.8028$)

TABLE 8
LONG-TERM CHANGES IN THE PROPERTIES OF DOUBLE-MODE STARS ACCORDING TO A REANALYSIS OF THE PHOTOMETRIC DATA.

Star	Band	JD-2400000	f_F d ⁻¹	A_F ^a mag	f_{1O} d ⁻¹	A_{1O} mag	$A_F/A_{1O}f_F/f_{1O}$	f_{mF} ^b d ⁻¹	f_{m1O} ^c d ⁻¹	Remark
V13	pg	23858-24684	2.07019	0.46						
	pg	28963-35933	2.07022	0.36						
	pg	36367-37131	2.07023	0.38						
	pg	39944-47554	2.08544	0.28						
	CCD V	50162-54965	2.08555	0.22	2.85123	0.07	3.3	0.7315	0.0069	0.0069
	CCD V	55935-56137	2.08555	0.23	2.85127	0.08	2.9	0.7314	0.0072	0.0072 f_{2O} , $f_{PD} = 3f_{2O}/2$, $f_{0.61}$
V44	pg	13372-22840	1.97485	0.58						
	pg	24283-30078	1.97490	0.54						
	pg	31965-36991	1.97471	0.45						
	pg	37018-47555	1.97470	0.30						
	CCD V	48755-49091	1.975:	0.22	2.7:	0.06	3.7	0.731:	0.019:	
	CCD V	50893-51257	1.9744	0.40					0.010/0.019	
	CCD V	54869-54965	1.9736	0.30	2.685:	0.03	10.0	0.7350	0.009/0.018	
	CCD V	55935-56137	1.98505	0.21	2.71650	0.06	3.5	0.7307	0.005/0.010/0.020	0.018
V68	pg	22455-24684	2.08989	0.25	2.80916	0.26	0.8	0.7440		
	pg	33034-39944	2.08985	0.22	2.80920	0.25	0.8	0.7439		
	CCD V	48755-50176	2.08985	0.17	2.80921	0.20	0.8	0.7439		
	CCD V	50893-51257	2.08988	0.17	2.80918	0.19	0.9	0.7439		
	CCD V	54869-54965	2.0899	0.17	2.8092	0.21	0.8	0.7440		
	CCD V	55935-56137	2.08986	0.17	2.80921	0.20	0.9	0.7439		$f_{0.61}$
V79	pg	13372-20656	2.0666	0.66						
	pg	22729-24684	2.0697	0.51						
	pg	28963-35933	2.0690	0.50					0.050:	
	pg	36367-39672	2.0691	0.44					0.012:	
	pg	41061-42970	2.0698	0.25						residual at f_F
	pg	43160-44760	2.0691	0.42						residual at f_F
	pg	44988-46566	2.0696	0.43						
	pg	47202-47329	2.0711	0.49					0.015:	signal at 2.087 d ⁻¹
	pg	47552-47686								at JD 2447620 pulsation ceases
	pg	47970-48485	2.0843	0.29						strong residual at 2.0743 d ⁻¹
	CCD V	48755 49091	2.0856:	0.11	2.7931:	0.16	0.7	0.7467		
	CCD V	50162-50176	2.084	0.09	2.792	0.17	0.5	0.746		
	CCD V	50893-50972	2.0865	0.09	2.7914	0.17	0.5	0.7474	0.015	
	CCD clear	54257-54273	2.072	0.32	2.795	0.13	2.5	0.741		
	CCD V	54869-54965	2.06913	0.41					0.016	largest mod. peak at 2.085 d ⁻¹
	CCD V	55935-56137	2.06914	0.35					0.006/0.016/0.055	3rd largest mod. peak at 2.085 d ⁻¹
V87	pg	22729-24684	2.08184	0.14	2.79737	0.32	0.5	0.7442		
	pg	33390-39944	2.08261	0.09	2.79732	0.25	0.4	0.7445		
	pg	42838-47555	2.08147	0.13	2.79733	0.29	0.5	0.7441		
	CCD V	48755-50176	2.08248	0.08	2.79732	0.20	0.4	0.7445		
	CCD V	50893-51283	2.08238	0.09	2.79738	0.20	0.5	0.7444		
	CCD V	54869-54965	2.08263	0.09	2.79715	0.22	0.4	0.7446		
	CCD V	55935-56137	2.08260	0.08	2.79729	0.19	0.4	0.7445		$f_{0.61}$
V99	pg	13372-15161			2.77062	0.42				
	pg	22455-24684	2.0673:	0.22	2.7712:	0.23	0.8	0.7460:		
	pg	33034-35933	2.0719	0.10	2.7712	0.21	0.5	0.7476		
	pg	36367-39933	2.0732	0.14	2.7711	0.20	0.6	0.7482		residual at 2.069
	pg	42838-43254			2.7711	0.20				
	CCD V	48755-49091	2.0746:	0.08	2.7707:	0.24	0.3	0.7488:		
	CCD V	50893-50972	2.0740	0.15	2.7696	0.10	1.4	0.7488		0.025
	CCD V	54869-54965	2.0734	0.10	2.7703	0.18	0.6	0.7484		0.025
	CCD V	55935-56137	2.0743	0.12	2.7691	0.09	1.4	0.7490	0.002:	0.025
V125	pg	13372-24684			2.85862	0.36				
	pg	28963-39944			2.85860	0.28				
	pg	42838-47555			2.85859	0.26				
	CCD V	48755-50176			2.85860	0.21				
	CCD V	50893-50972			2.85869	0.20				
	CCD V	54869-54965			2.85857	0.21				
	CCD V	55935-56137	2.1233	0.01	2.85862	0.21	0.03	0.7428		$f_{0.61}$
V166	CCD V	48755-50176	2.0619	0.18	2.8031	0.14	1.2	0.7356		strong residual at f_F
	CCD V	50893-51283	2.0621	0.20	2.8031	0.09	2.3	0.7356	0.014	
	CCD V	54869-54965	2.0623	0.23	2.8034	0.12	1.9	0.7356	0.014	
	CCD V	55935-56137	2.0617	0.20	2.8033	0.09	2.2	0.7355	0.014	0.023 $f_{PD} = 7f_F/2$
V200	CCD V	50893-51283	2.0606	0.19	2.7700	0.18	1.0	0.7439		
	CCD flux (V)	50920-50965	2.0606		2.7690		0.8	0.7442		
	CCD flux (B)	55935-56137	2.06058		2.76995		0.9	0.7439		
V251	CCD V	50896-51283	2.0998:	0.18	2.8232:	0.15	1.2	0.7438		
	CCD flux (B)	55935-56137	2.0998		2.8230		1.0	0.7438		
V252	CCD B	48755-50622	2.2071:	0.28	2.9754:	0.32	0.9	0.7418		
	CCD V	50893-51283	2.2076	0.18	2.9755	0.19	0.9	0.7419		
	CCD flux (B)	55935-56137	2.2075		2.9754		1.0	0.7419		

^aFourier amplitude; A_{pg} is about 1.3 times larger than A_V , at the earliest epochs (JD 2413000-2415000) it is about 2 times larger.

^bModulation frequency of the F mode.

^cModulation frequency of the 1O mode.

frequency ratio matches the required P_2/P_0 period-ratio range ($0.582 - 0.593$) of F+2O double-mode RR Lyrae-type stars (Moskalik 2014). Therefore, we identify this signal with the second radial overtone mode. The radial-mode frequencies are far from the $2f_{10} = f_F + f_{20}$ resonance condition, $2f_{10} - f_{20} = 2.1508$, while f_{10} is observed at 2.0855 d^{-1} .

Modeling the radial-mode frequencies of triple-mode stars provides a unique opportunity to determine the physical parameters of the stars precisely. Unfortunately, pulsation models fail to explain until yet the anomalous period ratios observed in M3 and in the Galactic Bulge (C04, Smolec 2015a), and V13 is one of the RRd stars with a too small period ratio. According to the D12, and the combined K98, B06 and J12 data, its $f_{10}/f_F = 0.7315$ frequency ratio is even smaller than that was derived by C04 (0.7379).

V13 was identified first as a double mode variable by C04. It had been classified as an RRab star in all the previous studies. The fundamental mode was the dominant mode in each available data set.

The pulsation period decreased drastically by 0.0036 d ($\Delta f_F = 0.015 \text{ d}^{-1}$) in the 1960s. The period change was accompanied by a 25% decrease of the fundamental-mode amplitude. Unfortunately, the quality of the photographic data does not make it possible to detect small-amplitude signals in these data. Therefore, it cannot be excluded that the strong period change and amplitude decrease were synchronous with the appearance of the overtone mode.

The reanalysis of all the CCD data shows that, in fact, the overtone mode is present in all these observations. The 0.015 d^{-1} frequency shift of the fundamental mode preceding the appearance of the 1O mode is about twice of the modulation frequency (0.0072 d^{-1}) detected in the CCD data. The f_F , $2f_F$, and f_{10} frequencies identified in the different data sets of V13 are shown schematically in Fig. 17.

7.2. The double-mode stars

V44 was classified as an RRab star with a complex light-curve variation assigned to a very strong Blazhko effect previously. Its 0.506-d fundamental-mode period is significantly longer than the $0.453 - 0.485\text{-d}$ period of the other RRd stars. There are 18 RRab stars in M3 that do not show the overtone mode, but their periods are shorter than the F-mode period of V44.

We find, however, that frequency around the possible position of the 1O-mode appears already in the C01 and J12 observations, not only in the D12 data. It is the highest amplitude signal in the residual spectra of these data sets, but 1O is not detected in the H05 and B06 observations. The amplitude of the fundamental mode is $1.5 - 2.0$ times larger in the H05 and B06 data than in those observations when the overtone mode is also present, though with a small amplitude.

The detected frequencies of V44 at the positions of f_F , $2f_F$ and f_{10} are shown for the available CCD data sets schematically in Fig. 18. The frequency of the main component at f_F is 0.011 d^{-1} larger (the period is 0.0029 d shorter) in the D12 data than it was previously. The frequency, which is identified as $f_F - f_m$ in the D12 data is the largest-amplitude signal in the 2009 observations and in all the other CCD data. Despite of the fundamental-

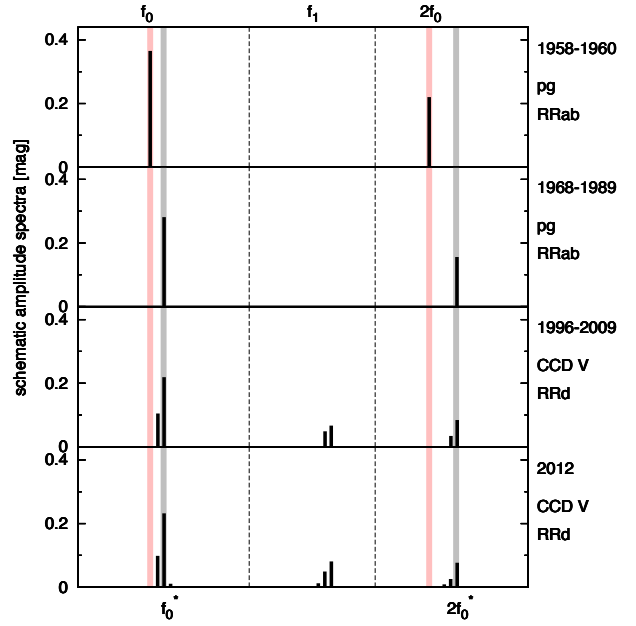


FIG. 17.— Evolution of the f_F , $2f_F$, and f_{10} frequency components of V13 in the 1958 – 2012 period. Pink and gray strips indicate the positions of the fundamental mode and its first harmonic detected at the different epochs.

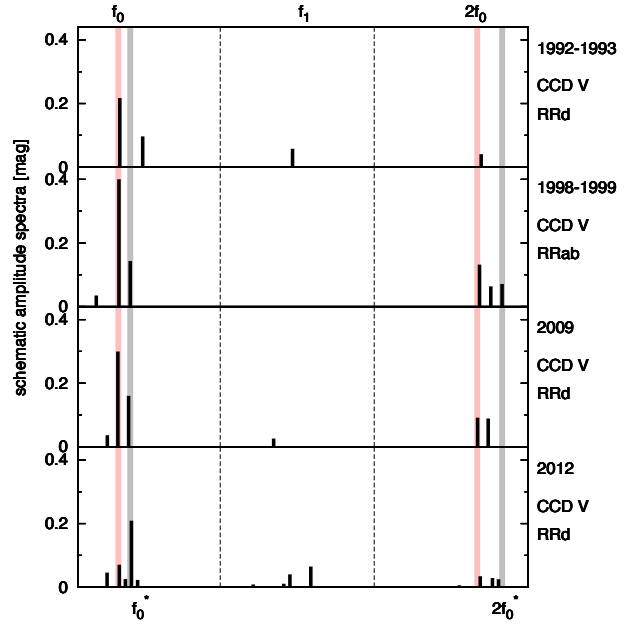


FIG. 18.— The same as Fig. 17 for the frequency components of V44 in the 1998 – 2012 period.

mode frequency having a significantly larger amplitude than the largest side frequency (the amplitude ratio is ~ 3) in the D12 data, $2f_0$ and $3f_0$ are not detected; only modulation components appear in the 2nd, and 3rd harmonic orders. In contrast, harmonic components of the F-mode frequency are detected up to harmonic orders $4 - 5$ in other CCD data.

The frequency ratio of the radial modes has an anomalously low value in each data showing double-

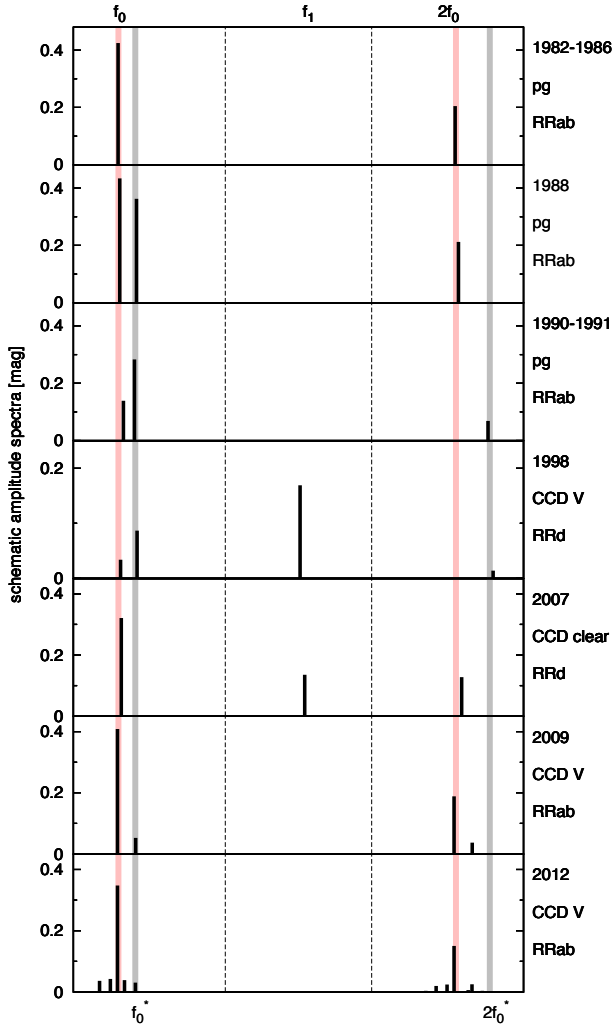


FIG. 19.— The same as Fig. 17 for the frequency components of V79 in the 1982–2012 period. Only those epochs are shown which give a substantially different solution.

mode pulsation.

V68 All the CCD and photographic data yield a very similar solution with $0.7439 - 0.7440$ frequency ratio and with a dominant overtone mode.

V79 Its unique behavior was discussed in several previous papers (e.g. Clement et al. 1997; Clement & Goranskij 1999; Clement & Thompson 2007; Goranskij et al. 2010). Data published in Clement & Goranskij (1999) and Goranskij & Barsukova (2007) are also utilized in the analysis besides the other CCD and photographic data.

V79 was a double mode star only in the 1992 – 2007 interval. Before and after this double-mode episode it had/had been an RRab star. A strong Blazhko effect has been detected since 2008. Indications of Blazhko modulation of the fundamental mode are evident in many parts of the photographic data, but no Blazhko period can be derived. For completeness, we also include a summary of the peculiar changes of V79.

During the major part of its double-mode stage and

TABLE 9
PERIOD AND MAGNITUDE CHANGES OF V79 BETWEEN 1992 AND 2012.

Year	P_0	$\langle B \rangle$		$\langle V \rangle$		$\langle I_C \rangle$		Data
		mag	int	mag	int	mag	int	
1992-1993	0.47948	15.95	15.94	15.67	15.66			C01
1996	0.4798			15.71	15.70			K98
1998	0.47927	16.00	15.99	15.72	15.71	15.28	15.28	H05,B06
1998	0.47927			15.71	15.70			CG ^a
2009	0.48329			15.78	15.72			J12
2012	0.48329	16.06	16.00	15.76	15.72	15.31	15.29	D12

^aClement & Goranskij (1999)

just preceding it (between 1990 and 1998) the fundamental mode frequency was 0.016 d^{-1} larger (the period 0.004 d shorter) than it was prior and after the double-mode episode. The amplitude of the overtone mode was about twice the amplitude of the fundamental mode in 1992 – 1998 while the fundamental mode has become the dominant mode again from 2006. The frequency ratio of the radial modes was larger in 1992 – 1998 and smaller than normal in 2006 – 2007 by about 0.002 .

The changes in the frequency spectrum of V79 is documented in Fig. 19. Only the substantially different solutions are shown. Preceding the emergence of the overtone mode, the fundamental mode frequency had been splitted into two components and the amplitudes of the components had been transposed. Besides the dominant frequencies observed at 2.0711 and 2.0843 d^{-1} , residual signal appeared at 2.087 and 2.0743 d^{-1} frequencies in the 1988 and 1990 – 1991 photographic data, respectively. The combined 1998 CCD data (H05, B06, and CG99) showed the dominant fundamental frequency at 2.0865 d^{-1} but a frequency component at 2.0719 d^{-1} appeared in this data set, too. The frequency changes of the first harmonic frequency of f_F followed the displacement of the fundamental frequency. After the double-mode episode, several $k f_F + f_m$ ($f_0 = 2.069$, $f_m = 0.016 \text{ d}^{-1}$) components appeared in the 2009 data. The frequency of the strongest modulation component equals with the fundamental-mode frequency (2.085 d^{-1}) observed during the double-mode phase. Later, the modulation has become more complex, multiple periodic with 0.006 and 0.016 d^{-1} frequencies of the primary and secondary modulations. The 2.085 d^{-1} frequency component appears in the D12 data, too. However, it is only the fourth largest modulation component with 0.031 mag amplitude.

Table 9 lists the fundamental-mode period and the magnitude- and intensity-averaged mean B, V, I_C magnitudes of V79 derived from the CCD data. According to the pulsation equation (given e.g. by Marconi & Degl’Innocenti 2007) a 0.0043 increase of $\log L$, i.e., a 0.011 mag decrease in magnitude, accounts for a 0.004 d increase of the pulsation period. Though the changes of the intensity-averaged mean magnitudes may support that luminosity changes are standing behind the period increase of V79, we have to note also that differences of the order of 0.01 mag in photometric results of GCs are common, simple because of calibration and/or instrumental effects. Moreover, the much larger decrease of the magnitude averaged brightness may be the consequence of that sinusoidal overtone-mode

was dominant in 1992 – 1998 but the large-amplitude, asymmetric RRab-type light curve observed in 2009 and 2012 is strongly nonlinear. Consequently, the observed increase of the intensity-averaged mean magnitudes may also arise, at least partly, from the differences of the light-curve shapes.

V87 All the CCD and photographic data yield a very similar solution with a dominant overtone mode. Period changes of both modes are marginal.

V99 The period ratio of the radial modes are anomalously large according to each observation that shows double-mode behavior. The reanalysis of the photographic data has revealed that besides the overtone mode the fundamental mode is also detectable in the majority of the data. Most probably, the improper phase coverage of the very-long-period Blazhko cycle of the fundamental mode (Sect. 6.1) explains the previously detected changes of the fundamental-mode frequency. The $\sim 0.001 \text{ d}^{-1}$ decrease of the overtone-mode frequency is probably real. Although the frequency of the $f_F + f_m$ component detected in the D12 data is very close to the exact 3:4 resonance position to the overtone mode ($(f_F + f_m)/f_1 = 0.7498$), the amplitude of this modulation component is only the half of the amplitude of $f_F - f_m$. Even in such close-resonance position, there is no sign that the 3:4 resonance would be in action (see also in Jurcsik et al. 2014).

V125 The fundamental mode is detected only in the D12 data. Its very small amplitude makes the detection of this component in the previous data unlikely. The period of the overtone mode is very stable. The star is not over-luminous and lies at the blue edge of the double-mode region.

V166 Double-mode behavior is observed in each CCD data set. Both the fundamental- and the overtone-mode light curves are modulated. The period ratio is anomalously small.

V200 Each CCD data show double-mode behavior. The H05, B06, and the D12 data yield a similar light-curve solution with similar amplitudes of the fundamental and the overtone modes and with a normal period ratio. The C01 observations of V200 are not publicly available. Based on the analysis of a reprocessed version of the C01 data, C04 listed V200 as one of the double-mode stars with a too small period ratio. However, the strong aliasing of the C01 data makes the frequency determinations of C04 dubious. The strongest alias frequencies appearing in the C01 observations are separated by 0.003, 0.006, and 0.009 d^{-1} . The differences between the f_F and f_{10} frequencies given by C04 and found in the other observations are -0.006 and 0.009 d^{-1} , which supports that, probably, no significant period change occurs in this star. No difference between the amplitude ratios of the fundamental and first-overtone modes larger than expected from the uncertainties is seen between the solutions of the H05, B06, and the D12 data. The amplitude changes of the modes found by C04 between the 1992 and 1993+1997 seasons are probably the consequence of the inadequacy of the data sampling and the mistaken frequency identification. Based on the characteristics of its light curve and period ratio, this star seems to be the twin of V68.

V251 The frequencies published by C04 and B06 are significantly different from the results obtained from the

D12 data. The reprocessed C01 data of V251 analyzed in C04 are not publicly available. The differences between the fundamental- and overtone-mode frequencies given by C04 and found in the present analysis are 0.009 and -0.003 d^{-1} . Thus, it seems that aliasing might have biased the frequency determination of V251 in C04, too. The frequencies of V251 given by B06 are $0.03 - 0.04 \text{ d}^{-1}$ larger than detected in the D12 data. Although the frequencies identified by B06 are indeed the largest amplitude signals in the Fourier spectrum of the data, a solution with frequency values close to the results of the D12 data fits the observations with 7% smaller residual scatter. C04 reported significant amplitude changes of the modes, however, the D12 data do not show any frequency in the residual spectrum at the radial modes indicating that the light curves of the modes are stable.

V252 The C01 observations of V252 are not publicly available. The combined H05 and B06 data yield a similar solution as the D12 data.

Our analysis shows that in normal-period-ratio RRd stars of M3 (V68, V87, V125, V200, V251, and V252) either the overtone mode is dominant or the fundamental mode has a similar amplitude as the overtone. The majority of the known RRd stars share this property. Based on the results given in Table 8, the modal content, i.e., the frequencies, and the amplitude- and period-ratios of V68 and V87 have been stable during the century covered by the observations. Most probably, V125, at the blue border of the double-mode region, is in the stage of the fall-off of the weak fundamental-mode component. This is supported by the similarity of the periods, amplitudes, colors and the light curves' shapes of V125 and V126 (the longest-period not over-luminous RRc star in M3). We suppose that the differences between the amplitude ratios and the frequencies published in C01, C04, H05 and B06 and obtained from the D12 data for V200, V251, and V252 do not reflect real changes of the double-mode properties of these stars, but arise from the strong aliasing problems of the previous data, which may have biased the previous frequency determinations. We conclude that the pulsation properties of double-mode stars with normal period ratio are stable, they do not change significantly on a timescale of about a century in M3.

8. DISCUSSION ON THE PROPERTIES OF BLAZHKO RRD STARS

RR Lyrae stars showing Blazhko effect were first reported by Soszyński et al. (2014). A detailed analysis of these special variables found in the OGLE Galactic bulge collection were published recently by Smolec (2015a). It is also probable that long-period Blazhko modulation accounts for the detected changes of the amplitude ratios of the radial modes of NSVS 5222076 (Hurdis & Krajci 2012). The Blazhko modulation of 10/20 double-mode Cepheids were discussed in detail by Moskalik & Kołaczowski (2009).

Modulation is observed among 10/20 LMC Cepheids either in both modes or neither of them (Moskalik & Kołaczowski 2009). The same is true for F/10 double-mode RR Lyrae stars in M3. However, despite of the large amplitude modulations of the F and 10 modes, the small-amplitude 20 mode of V13 is not

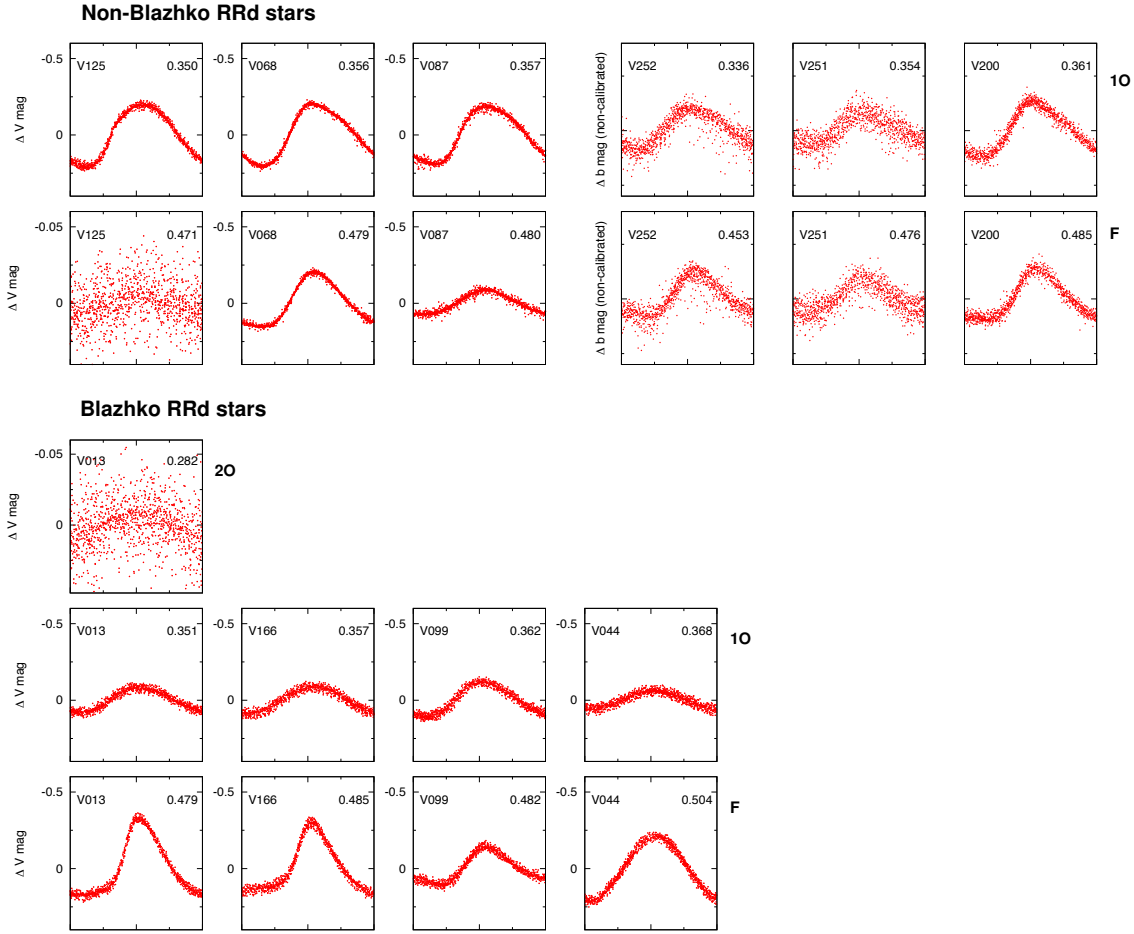


FIG. 20.— Phased light curves of the radial modes of Blazhko and non-Blazhko double-mode stars. The periods (d) are given in the top-right-side of the panels. The data are prewhitened for all the other signals.

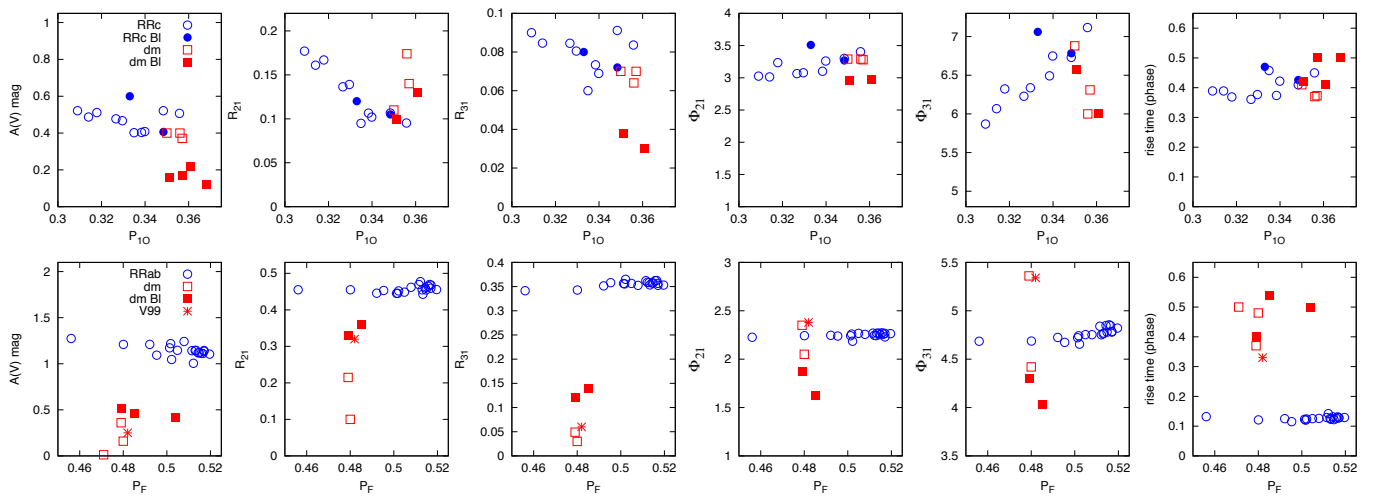


FIG. 21.— The V light-curve parameters (peak-top-peak V amplitude, Fourier amplitude ratios and phase differences, and the rise time) of the first-overtone (top panels) and the fundamental (bottom panels) modes of double-mode stars and of similar-period single-mode RRc and RRab stars are compared. The fundamental-mode parameters of V99 are denoted by a different symbol, because these parameters might not be reliable as only about half of the modulation cycle of the fundamental mode is covered by the observations.

modulated.

Moskalik & Kołaczowski (2009) explained the anti-phase modulations of the radial modes of Cepheids by non-stationary resonant coupling of one radial mode with the another radial or a non-radial mode, and with the sharing on the excitation sources between the two radial modes. Resonances are in fact detected in Cepheids, e.g. a 2:1 resonance produces well-seen features of the light curve of bump-Cepheids. Therefore, we have checked whether the light curves of the radial modes of the Blazhko double-mode stars show any systematic difference from the light curves of other double-mode stars and from single-mode RR Lyrae stars. Fig. 20 shows the light curves of the radial-modes of double-mode stars. The data were prewhitened for the Fourier components of the other mode, for the modulation of both modes and for the linear-combination terms. The only conspicuous difference between the light curves of the Blazhko and non-Blazhko double-mode stars is that the overtone mode is the dominant mode (or the modes have similar amplitudes: V200, V251, V252) in RRd stars without light-curve modulation, while the fundamental mode has larger amplitude than the overtone in Blazhko RRd stars.

There is no well seen distortion in the mean light curves of the radial modes of Blazhko stars (see in Fig. 20). The fundamental-mode light curves of both Blazhko and non-Blazhko RRd stars differ from the light curves of single-mode RRab stars systematically, however, the lack of the higher-order harmonic components in RRd stars may explain the differences mostly.

None of the light curves at different Blazhko phases shown in Fig. 15 indicates that any resonance would affect their shapes either. Amplitude and phase modulations of both modes of the Blazhko double-mode stars are evident, however the light curve is smooth in each Blazhko phase. Only the fundamental mode of V44 displays irregularities, but this is produced by the complexity of the modulation probably.

The V light-curve parameters of the fundamental and the first-overtone modes of double-mode stars and of similar-period single-mode stars are shown in Fig. 21. The period, peak-to-peak V amplitude, the R_{21} , R_{31} , φ_{21} , φ_{31} Fourier amplitude-ratios and phase-differences and the rise-time (in phase units) are plotted versus the overtone- and fundamental-mode periods in the panels. The Fourier amplitude-ratios and phase-differences are not shown for the most sinusoidal components of the double-mode stars. The only systematic difference between the light-curve parameters of the overtone-mode of Blazhko and non-Blazhko RRd stars is that the A amplitudes and the R_{31} amplitude-ratios and maybe the φ_{21} phase differences of Blazhko RRd stars are systematically smaller than those of normal RRd and RRc stars. The rise time of each RRd star is around the same as the rise time of RRc stars (between 0.4 and 0.5). The amplitudes and amplitude-ratios of the fundamental mode of both Blazhko and non-Blazhko RRd stars are significantly smaller than those in normal RRab stars (the amplitude ratios of normal RRd stars are even smaller than of Blazhko RRd stars); the phase-differences of Blazhko RRd stars are also too small, while the phase-differences of normal RRd stars seem to be scattered around the values of RRab stars. The rise time of the fundamental mode of each RRd star is significantly larger than the

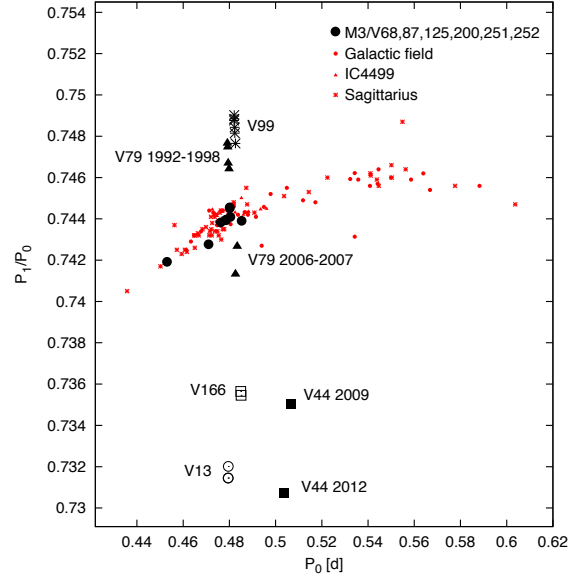


FIG. 22.— Petersen diagram, P_1/P_0 period ratio vs P_0 , of M3 double-mode stars is shown. Filled circles denote RRd stars with normal period ratio (V68, V87, V125, V200, V251, and V252). Other symbols denote the results obtained for Blazhko RRd stars at the different epochs according to the data given in Table 8. For comparison, the period ratios of double-mode stars in the Galactic field, IC 4499 and the Sagittarius dwarf galaxy are also shown.

rise time of RRab stars.

Based on the light curve’s shapes and the Fourier parameters of the Blazhko RRd stars we have not found any clear evidence of a distortion caused by resonance interaction.

In Jurcsik et al. (2014) we have shown that the anomalous period ratio is a common property of RRd stars showing Blazhko modulation of the radial modes in M3. The opposite is also true, none of the RRd stars with normal period ratio shows modulation of any of the modes. A similar tendency has been detected also in the Galactic bulge data (Smolec 2015a). The anomalous period ratios of RRd stars cannot be interpreted by any canonical explanation as it was discussed in detail in C04 and in Smolec (2015a).

In Fig. 22, the P_{10}/P_F period ratio of M3 double-mode stars are shown in comparison with results of the double-mode stars in the Galactic field, the IC 4499 globular cluster and the Sagittarius dwarf galaxy (compilation by McClusky 2008; Wils 2010; Walker & Nemec 1996; Cseresnjes 2001). The period ratios of six RRd stars (V68, V87, V125, V200, V251, and V252) fit the region covered by the data of the other systems. The period ratios of Blazhko RRd stars (V13, V44, V79, V99, and V166) are anomalous according to each of the data, which show double-mode behavior (see Table 8).

The frequencies appearing at the positions of f_F and f_{10} of RRd stars in the D12 data are shown in Fig. 23. Contrary to the single signals of the radial modes of normal period-ratio RRd stars, the frequency spectrum of Blazhko RRd stars are comprised from multiplets at the fundamental- and the overtone-mode frequencies. The highest-amplitude frequency belongs to the fundamental

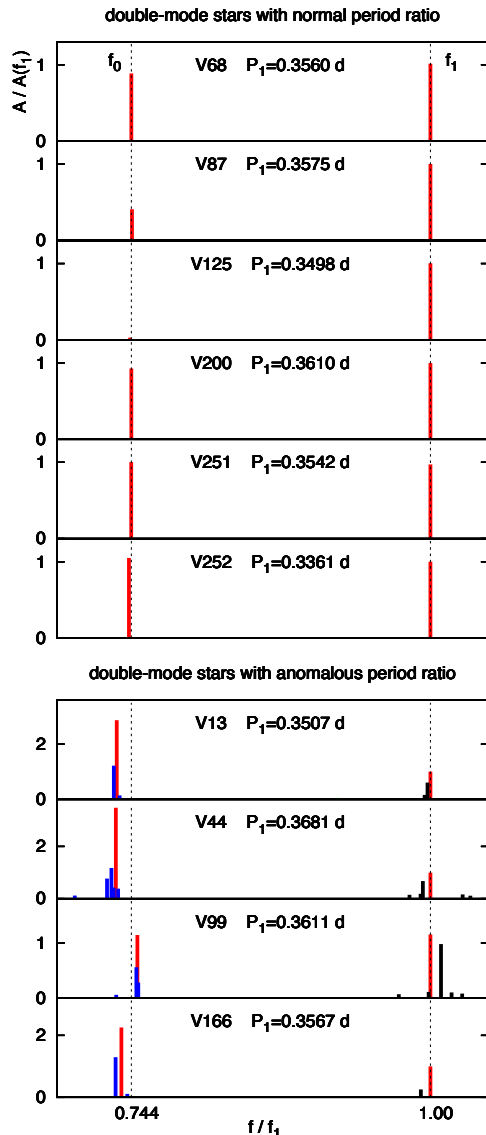


FIG. 23.— The fundamental- and overtone-mode frequencies of RRd stars in M3. The radial-mode frequencies of RRd stars with normal period ratio ($P_1/P_0 = 0.742 - 0.745$) are separate, single signals. On the contrary, complex multiplets at the radial-mode frequencies, characteristics of Blazhko modulation, are detected in RRd stars, which have anomalous period ratios.

mode in these stars.

The data given in Table 8 and Figs. 17, 18 and 19 indicate that in Blazhko RRd stars not only the period ratios are anomalous, but from time to time, a different component of the multiplets is the dominant component. OGLE-BLG-RRLYR-02530 (Smolec 2015a) seems to be a similar case. We have also checked whether any pair of the components of the multiplets would yield a period ratio in the required, 0.743 – 0.744 range. However, the mystery of the anomalous period ratios of Blazhko RRd stars cannot be resolved in this way; none of the components of the multiplets can give a proper period ratio.

Connected to the switch between the single- and double-mode states (just preceding or following it), as large as $0.010 - 0.015 \text{ d}^{-1}$ jumps of the fundamental-mode frequency ($0.003 - 0.004 \text{ d}$ jumps in the periods)

are detected in the Blazhko RRd stars, V13, V44, and V79. The frequency shift preceding the switch to double-mode stage is always positive (the period is decreasing) and it is accompanied by a strong amplitude decrease of the fundamental mode in all these stars.

Blazhko RRab stars are known to show large, irregular, rapid period changes (J12), however, all these changes are at least about an order of a magnitude smaller than the period changes observed in Blazhko double-mode stars. The switch from double-mode to fundamental-mode pulsation of OGLE-BLG-RRLYR-12245 (Soszyński et al. 2014) was accompanied by a 0.0005 d period increase (-0.003 d^{-1} in frequency). The period change was attributed to a nonlinear shift caused by the four-times increase of the amplitude of the fundamental mode by Soszyński et al. (2014). Although simultaneously with the appearance of the overtone mode, amplitude decrease of the fundamental-mode of V13, V44, and V79 are also detected, as their period shifts are about an order of a magnitude larger than in OGLE-BLG-RRLYR-12245, therefore, nonlinearity cannot explain the observed frequency shifts in these stars completely.

The displacement of the frequencies corresponds to the modulation frequency (half of it in V13) indicating that, instead of real frequency changes, the modulation is so strong that the side frequency has the largest amplitude in these cases. The dominant component is the $f + f_m$ frequency in the double-mode stage of these three stars. Thus, the anomalously low period ratios would become even lower if the pre-RRd frequency of the fundamental-mode were taken as the real fundamental-mode frequency in V13 and V44.

Anomalous period ratios are detected mostly (probably exclusively) in Blazhko RRd stars. Assuming that pulsation models give correct results for the period ratios of RR Lyrae stars (as in the case for non-Blazhko RRd stars) we have to think that the stellar structure of Blazhko and non-Blazhko stars should be different in order to explain the anomalous period ratios of Blazhko RRd stars.

The period ratios of the modulations of the overtone and the fundamental modes (P_{m10}/P_{mF}) of the double-mode Blazhko stars (V13, V44, V99, and V166) match the 1:1, 4:7, 1:10, and 3:5 small-integer ratios within about the error limits of the periods. The modulation-period ratios of some of the Galactic bulge Blazhko RRd stars are also close to small integer ratios (e.g. it is close to 3:10, 2:3, 1:3, 4:7, 1:5, 2:3 in OGLE-BLG-RRLYR-04598, -07393, -10728, -11311, -22356, and -32462, respectively).

It was already shown that the light curves of some Blazhko RRab stars were modulated with different periods either simultaneously or at different epochs and that the ratios of the modulation-periods of multiple-periodic Blazhko stars were close to the ratios of small integer numbers (Sódor et al. 2011; Jurcsik et al. 2011, 2012b; Benkő et al. 2014). In other stars, different components of the multiplets reflect the dominant modulation at different epochs, i.e., the amplitudes of the multiplet components vary in time, according to the long-term data as e.g. in V44, V140/M3 (this paper) and V2/M5 (Jurcsik et al. 2011). In Jurcsik et al. (2011), we wrote

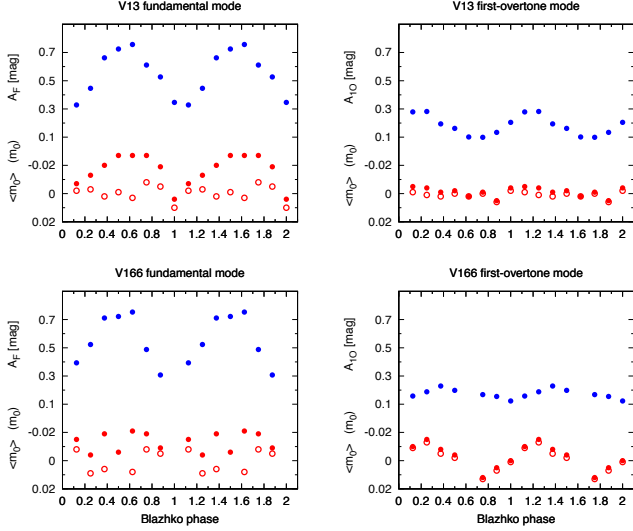


FIG. 24.— Variations of the fundamental- (left-side panels) and overtone-mode (right-side panels) amplitudes and the magnitude- and intensity-averaged mean V magnitudes ($\langle m_0 \rangle$, and $\langle m_0 \rangle$) shown by open and filled circles, respectively) of the light curves of V13 (top panels) and V166 (bottom panels) during the Blazhko cycle. Note that the Blazhko periods of the fundamental and the overtone modes are the same for V13, but they are respectively 71.5 d and 43.5 d for V166. The results are derived for the light curves in the eight Blazhko phases shown in Fig. 15. The light curves are prewhitened for the pulsation and modulation of the other radial mode and for the linear-combination terms.

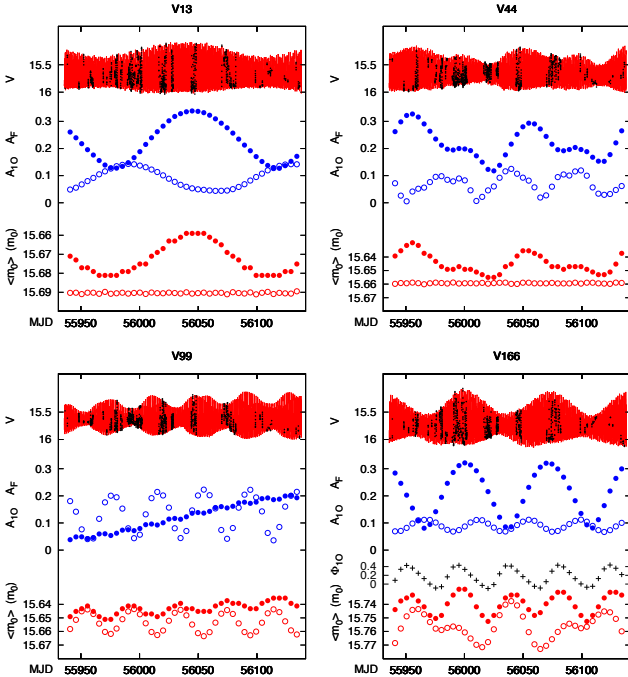


FIG. 25.— Amplitudes (A_{1O} and A_F shown by open and filled blue circles, resp.) and magnitude- and intensity-averaged mean brightness (shown by open and filled red circles, resp.) of Blazhko RRd stars derived from 5-d segments of the synthetic data generated from the full light-curve solutions. In the top part of each plot the synthetic light curves are shown with the observations over-plotted. The phase variation of the overtone mode of V166 is also shown by ‘+’ symbols.

‘If these modulations correspond to elements of series of equidistant spacing multiplets, their frequency ratios are close to fractions of small-integer numbers. This indicates that the observed modulation frequency values may not always correspond to the ‘base frequency’, i.e., the shortest frequency component of a multiplet’. The ratios of the modulation periods of the F and 1O modes in Blazhko double-mode stars may be explained similarly. If both modes select their modulation frequency from the same, rich multiplet spectrum, than any small-integer ratio of the modulation periods is equally possible.

Smolec (2015a) found a ‘feedback effect’ between the star’s brightness and pulsation-mode amplitude in some of the OGLE Blazhko RRd stars. They noticed that: ‘larger the amplitude of the dominant mode, smaller the mean stellar brightness’.

The connection between the mean brightness and amplitude variations is checked both on the observed and on synthetic data. The magnitude- and intensity-mean V brightnesses and the amplitudes of the F and 1O modes are shown in Fig. 24 as derived from the disentangled light curves of the radial modes of V13 and V44, the two Blazhko RRd stars which have good pulsation-phase coverage of both radial modes in most of the Blazhko phases. The same parameters are also determined by fitting the radial-mode frequencies and their linear combinations to 5-d segments of synthetic data of the 4 Blazhko RRd stars (Fig. 25). The synthetic data are generated from the modulated radial modes’ components of the full light-curve solutions.

Mean-brightness variations are indeed detected in each of the four Blazhko RRd stars. The intensity-averaged mean brightness differ significantly from the magnitude-averaged value in some cases, because of the non-sinusoidal shape of the light curves, which is especially true for the fundamental mode in the large-amplitude phase of the modulation. If we accept that the intensity-averaged mean-brightness variation reflects the luminosity changes, as it was shown in Blazhko stars (e.g. in Jurcsik et al. 2008; Sódor et al. 2011; Jurcsik et al. 2012b) using the photometric Baade-Wesselink method (Sódor et al. 2009), the brightness variations of V13 and V44 are in phase with the amplitude variations of the dominant fundamental mode. The intensity-averaged mean-brightness variation of V99 is twofold, it varies in anti-phase with the amplitude changes of the overtone mode, and, meanwhile, a gradual brightness increase, parallel with the amplitude increase of the fundamental mode, is also observed. The amplitude changes of V166 are the most surprising: the intensity-averaged mean brightness does not show any systematic changes with the modulation period of the dominant fundamental mode, which shows large-amplitude modulation, instead, it varies with the modulation period of the overtone mode. The brightness variation is in a $\pi/2$ phase lag with the very small-amplitude changes of the overtone mode. Actually, the intensity-averaged mean brightness follows the phase variation (also shown in Fig. 25 for V166) of the overtone mode. As a summary, no universal connection between the brightness and the amplitude variations seems to exist.

9. SUMMARY

The light variation of overtone and double-mode RR Lyrae stars has been investigated based on an extended new BVI_C time-series photometry and on archive data. The GC data has a unique potential to map the distribution of variables showing different properties and extra features in a large and homogeneous sample of stars.

Peculiarities of the behavior of stars showing a frequency component at 0.61 frequency ratio, the Blazhko effect, period doubling, and double-mode pulsation are discussed in all details.

The most important findings of our study are summarized in the followings.

1. The $f_{0.61}$ frequency is observed in 14 RRc stars, three normal and one anomalous period-ratio RRd stars. Sect. 4.
2. The amplitude and frequency of the $f_{0.61}$ component show considerable changes, while the radial mode component remains stable. Sect. 4.1.
3. The light curves of RR_{0.61} stars are sinusoidal, with a reduced, if any, bump preceding maximum brightness. Sect. 4.2.
4. The $f_{0.61}$ frequency occurs in 4 RRd stars, in each RRc star at the blue-side of the double-mode region, and in some bright, evolved variables with periods longer than the period of the 1O-mode of RRd stars. Actually, it is detected in each RRc star with $B - V \gtrsim 0.24$ mag and $V - I \gtrsim 0.36$ mag color indices. Connection between the double-mode pulsation and the appearance of the $f_{0.61}$ frequency is proposed based on the distribution RR_{0.61} stars. Sect. 4.3.
5. Period doubling of the small-amplitude 2O mode of V13 ($3f_{2O}/2$) and the fundamental mode of V166 ($7f_F/2$) are detected. Blazhko effect of both the F and 1O modes of V13 and V166 are observed, but the 2O mode of V13 does not show any modulation. No half-integer component of any first-overtone or $f_{0.61}$ frequency has been detected. Sect. 5.
6. Blazhko modulation of both radial modes of four RRd stars, and three luminous RRc stars are observed. A small-amplitude frequency component appears in the residual spectra of one normal-brightness RRc star and in a 2O variable close to the overtone-mode frequency. Strong phase modulation characterizes the modulation of the overtone modes. Sect. 6.
7. Two new double-mode stars are identified: F-mode frequency is detected in the RRc star, V125, and 1O-mode frequency in the RRab star, V44. Sect. 7.
8. The most complex variability is detected in V13, which shows three radial modes, strong Blazhko effect of the F and 1O modes, period doubling of the 2O mode, and the $f_{0.61}$ frequency component, as well. Although modeling the radial-mode frequencies of triple-mode stars may yield accurate physical parameters, unfortunately, this is not the case in V13. Pulsation models fail to explain the anomalous period ratios of some RRd stars observed in

M3 and in the Galactic bulge (C04, Smolec 2015a), and V13 is one of the Blazhko RRd stars with anomalously small P_{1O}/P_F period ratio. Sect. 7.1.

9. The modal content and the frequencies of RRd stars seems to be remaining stable during the timescale of the photometric observations of M3 for variables with period ratios matching the sequence defined by the majority of the known RRd stars on the P_{1O}/P_F - P_F Petersen diagram (V68, V97, V200, V251, and V252). Sect. 7.2.
10. The period-ratio of four RRd stars (V13, V44, V99, and V166) are anomalous, and it was the case for V79 during its double-mode stage, as well. To explain the anomalous period ratios of Blazhko RRd stars is a great challenge for stellar pulsation theory. The anomalous period ratios of Blazhko RRd stars can, in principle, be explained by extreme high- or low-metallicity models (see fig. 7. in Smolec 2015b), however, it is unlikely in about 50% of the RRd stars in M3, as no extreme metallicity value of any other star of the cluster has been reported. Each anomalous period-ratio RRd stars shows Blazhko modulation of both the F and 1O radial modes. The fundamental mode is dominant in these star. Sects. 7.2, 8.
11. No evidence of a resonance-induced systematic deformation of the light curves of Blazhko RRd stars is found, still in the case of V13, where the modulations of the F and 1O modes have the same periods and they show anti-phase amplitude changes similar to 1O/2O Cepheids. Sect. 8.
12. The M3 data show, that the ‘feedback effect’ (Smolec 2015a) between the mean brightness and the amplitude of the dominant mode in Blazhko RRd stars does not exist in general. The connection between the amplitude variations of the modes and the mean brightness (defined as the intensity-averaged mean V magnitude) is different star by star. Sect. 8.
13. Large period jumps are recorded in V13, V44, and V79, which are connected to the switch from fundamental- to double-mode pulsation. The value of the frequency increase equals with the modulation frequency (or its half), and strong amplitude decrease accompanies the appearance of the overtone mode in Blazhko RRab stars. Different components of the Blazhko multiplets seem to be the highest amplitude signals at the fundamental mode frequency in single- and double-mode stages in these stars. Sects. 7.2, 8.
14. The period-ratio anomaly of Blazhko RRd stars cannot be resolved with any pair of the components of the multiplets appearing at the radial modes. Sect. 8.

As a conclusion we would like to put into the focus the strange $f_{0.61}$ signals detected in different samples of classical radial pulsating stars and the special anomalous period ratios of RRd stars observed in this M3 survey as open questions requiring further investigation.

ACKNOWLEDGMENTS

The authors would like to express their acknowledgement to the referee for the notes, questions and comments, which helped us to improve the clarity and quality of the paper. GH acknowledges support by the Ministry for the Economy, Development, and Tourism's ssyPrograma Iniciativa Científica Milenio through grant IC210009, awarded to the Millennium Institute of Astrophysics; by Proyecto Basal PFB-06/2007; by Fondecyt grant 1141141; and by CONI-

CYT Anillo grant ACT1101. KK is supported by a Marie Curie International Outgoing Fellowship within the 7th European Community Framework Programme (PIOF 255267 SAS-RRL). NJ acknowledges the OTKA K-83790 grant, MA the grant of the MTA CSFK Lendület Disk Research Group, AP the LP2012-31, K-109276 and K-104607 grants. KS has been supported by the Lendület-2009 program of the HAS, the Hungarian OTKA grant K-104607 and the ESA PECS Contract No. 4000110889/14/NL/NDe.

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